1

Introduction

1.1 OVERVIEW

A *machine* is a device consisting of interrelated components that modifies force or motion. *Machine design* is the art of planning new or improved machine(s) to accomplish specific purpose. The aim of design process is to shape and size the elements so that the resulting system can be expected to perform the intended function without failure.

Machine design may also be defined as the use of scientific principles, technical information and imagination in the design of mechanical structure, machine or a system to perform prescribed functions with maximum economy and efficiency. Design is an iterative process with many interactive phases so as to produce better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation.

1.1.1 Classification of Machine Design

The machine design may be classified as follows.

- *Adaptive design*: Here the designer makes minor alterations to the existing design and as such does not require special skills.
- *Development design*: Here the designer starts from the existing design but the final product may vary markedly from that of original design.
- *New design*: This type of design is entirely new but based on existing scientific principles. No scientific invention is involved but requires creative thinking to solve a problem.

1.1.2 Types of Design Based on Methods

- **Rational design:** This is based on determining the stresses and strains of components and thereby deciding their dimensions.
- **Empirical design:** This is based on empirical formulae which in turn are based on past experience and experiments.
  For example, when we tighten a nut on a bolt the force exerted or the stresses induced cannot be determined exactly but experience shows that the tightening force may be given by $P = 2840d$, where d is the bolt diameter in mm and P is the applied force in newton. There is no mathematical backing of this equation but it is based on observations and experience.
- **Industrial design:** These are based on industrial considerations and norms, viz. market survey, external look, production facilities, low cost, use of existing standard products.

- **Optimum design:** It refers to design of a component under a set of specified constraints.
- **System design:** It refers to design of orderly combination of parts in a complex mechanical system into a whole.
  Example: Automobiles.
- **Element design:** It refers to design of any element/component of the mechanical system.
  Example: Piston, crankshaft, connecting rod, etc.
- **Computer aided design:** It refers to the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

1.2 BASIC PROCEDURE IN MACHINE DESIGN

The basic procedure of machine design consists of a step by step approach from given specifications of functional requirement of a product to the complete description in the form of blueprints of the final product. It is to be noted that the process is neither exhaustive nor rigid and will probably be modified to suit individual problems. It can be outlined by flow diagrams with feedback loops as shown in **Fig. 1.1**.

Identification of Need

- Design process begins with recognition of the need, real or imagined, and a decision to do something about it.
  Example: The need for present equipment requires improving durability, efficiency, weight and cost.

Definition of the Problem

- It includes all the specifications for the object that is to be designed.
- Specifications include all forms of input and output quantities must be spelled out. It also includes the definitions of the members to be manufactured, cost, the expected life, the range, the operating temperature, and reliability.
- Once the specifications have been prepared, relevant design information is collected to make a feasibility study.
- As a result of this study, changes are made in the specifications and requirements.

Synthesis

- After defining the problem, now finding a solution and putting together is the most challenging and interesting part of design.
- This is sometimes called the iteration and invention phase (where the largest possible number of creative solutions is originated).
- The designer combines separate parts to form a complex whole of various new and old ideas and concepts to produce an overall new idea or concept.

Analysis and Optimization

- Analysis refers to finding out whether the system satisfies the requirements.
- Optimization refers to the best performance given by the system for which it is designed.

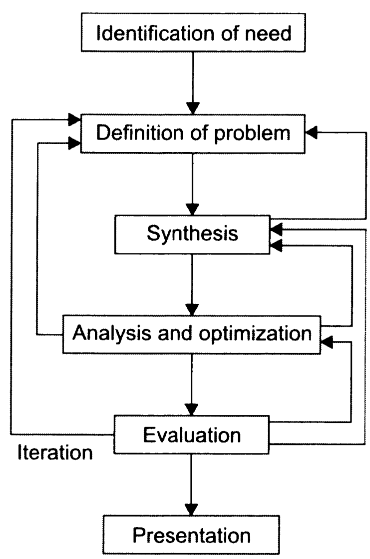


Fig. 1.1: Basic procedure in machine design

- If the design fails, the synthesis procedure must begin again till optimum performance is achieved.
- After synthesizing the system, the designer must specify the dimensions, select the components and materials, and consider the manufacturing cost, reliability, serviceability, and safety.

Evaluation

- This step is the final proof of a successful design and usually involves the testing of a prototype in the laboratory.
- Here we discover whether the design really satisfies the needs and other desirable features.

Presentation

- It refers to communicating the design to others.
- It is a selling job. The engineer, when presenting a new solution, attempts to prove that this solution is a better one.

1.3 ENGINEERING MATERIALS AND THEIR MECHANICAL PROPERTIES

Engineering materials are classified as follows.

1. Metals and alloys
   Metals are further classified as:
   a. Ferrous metals
   b. Nonferrous metals
2. Nonmetals.

Metal is defined as a substance which is solid at room temperature, has good mechanical properties (high strength, toughness, ductility, etc.) along with good thermal and electrical conductivity.

- Metals which contain iron as the main constituent are referred to as *ferrous metals.*
  Example: Cast iron, wrought iron, steel.
- Metals which do not contain iron as their main constituent are referred to as *non-ferrous metals.*
  Example: Aluminium, copper, brass, tin, etc.
- An alloy is a combination of two or more metals of which one is the base metal.

1.3.1 Selection of Materials for Engineering Purposes

The selection of proper material(s) for a given application is based on the following factors.

- Availability of the materials
- Suitability of the materials for the working conditions in service
- The cost of the materials.

1.4 MECHANICAL PROPERTIES OF METALS

Following are some of the most important mechanical properties of engineering materials.

- **Elasticity** is the ability of a material to regain its original shape and size, when the load causing the deformation is removed. *Example:* Steel, copper, aluminium, concrete, etc.
- **Plasticity** is the property of a material which does not regain its original shape and size, when the load causing the deformation is removed.
- **Strength** is the ability of a material to withstand the action of external forces.

- **Toughness** is the property of a material which absorbs energy elastically before failure, i.e. to withstand elastic and plastic deformation.
- **Stiffness/rigidity** is the ability of a material to resist elastic deformation.
- **Hardness** is the property of a material which resists indentation and/or penetration, or it is the property of a material to resist plastic deformation.
- **Resilience** is the property of a material to absorb energy elastically.
- **Ductility** is the property of a material of drawing it into wires. *Example:* Mild steel (MS)
- **Brittleness** is the property of the material by the virtue of which it breaks easily into pieces. *Example:* Cast iron
- **Malleability** is the property of the material by the virtue of which it can be rolled into sheets. *Example:* MS, aluminium, etc.
- **Creep** refers to the slow and progressive deformation of a material with time at constant stress.
- **Fatigue** refers to the behavior of a material when subjected to fluctuating or repeated loads.
- **Fracture** refers to the separation of a material into two/more parts under stress.
  - *Ductile fracture*: This occurs after extensive plastic deformation and is characterized by slow crack propagation.
  - *Brittle fracture*: This occurs by rapid propagation of a crack after a little or no plastic deformation.

1.5 FACTORS INFLUENCING MACHINE DESIGN/DESIGN CONSIDERATIONS

These refer to the characteristics which influence the design of element of the entire system. Following are some of the characteristics to be considered.

- **Strength:** This is the most important consideration while designing a new machine and its components. The machine should be sufficiently strong so that there is no damage or permanent deformation during its service life.
  The most commonly used proverb in designing a component is ***"sacrifice everything but not strength"***.
- **Stiffness or rigidity:** Under the effects of applied load, the machine should be rigid enough so that elastic deformation of the component is within the specified limits.
- **Availability of material:** Materials selected should be available easily at the lowest possible costs.
- **Cost:** Cost is the major factor to be considered while designing the machine and its components. The shape and material of the machine selected should be such that it can be produced with minimum labor and processing methods. The best design is the one which helps to get the finished product at the lowest possible cost.
- **Wear resistance:** Wear is defined as the damage to a surface that generally involves progressive loss of material and is due to relative motion between that surface and a contacting substance or substances. Wear resistance of the machine elements can be increased by increasing strength and hardness of the working surfaces. The wear of friction parts can be reduced by proper lubrication.
- **Reliability:** It is defined as the ability of the machine and/or component to perform its intended function without failure under stated conditions for a stated period of time. The reliability of the machine is very important for its proper functioning.
- **Light weight and minimum dimensions:** The machine elements should be strong, rigid and wear resistant with minimum weight and least dimensions. This can be achieved by:

- Employing light weight rolled sections.
- Employing latest methods of surface hardening.
- Using high strength grades of cast iron and light alloys.
- Introducing nonmetallic materials to replace ferrous and nonferrous materials.
- Improving the design of machine elements.

- **Safety:** For the safety of the operator from the machine, the hazard producing elements from the machine should be eliminated and the design should confirm to the safety codes.
- **Use of standard parts:** As far as possible, standard parts should be used in the design of the machine. This will help reduce the cost of the machine and ensure easy replacement of the damaged parts, etc.

1.6 CODES AND STANDARDS

A ***standard*** is a set of specifications given for materials, parts or processes so as to achieve uniformity, efficiency, and quality. The purpose of a standard is to limit the number of items in the specifications, thereby providing a reasonable inventory of tooling, sizes, shapes and varieties.

A ***code*** is a set of specifications for the analysis, design, manufacture, and construction of items. The purpose is to achieve a specified degree of safety, efficiency, and quality.

The most commonly used standards include:

- Standards for materials, their chemical composition, mechanical properties and heat treatment.
- Shapes and dimensions of commonly used elements, viz. bolts, rivets, nuts, chains belts, bearings, etc.
- Standards for fits, tolerances and surface finish of components.
- Standards for pressure vessels, boilers, overhead cranes, wire ropes, etc.
- Standards for engineering drawings of components.

Following are some of the standard organizations. The name of the organization provides a clue to the nature of the standard or code.

- American Bearing Manufacturers Association (ABMA)
- American Gear Manufacturers Association (AGMA)
- American National Standards Institute (ANSI)
- American Society of Mechanical Engineers (ASME)
- American Society of Testing and Materials (ASTM)
- International Standards Organization (ISO)
- National Association of Power Engineers (NAPE)
- National Institute for Standards and Technology (NIST)
- Society of Automotive Engineers (SAE), etc.

1.7 INDIAN STANDARDS FOR DESIGNATION OF MATERIALS (BIS SYSTEM)

... APPENDIX I/ Pg 453, DHB

1.7.1 Steels Designated on the Basis of Mechanical Properties (Letter Symbols)

The designation consists of the following in order.

i. Symbol 'Fe' or 'FeE' depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength.

ii. Figure indicating the minimum tensile strength or yield stress in N/mm^2.
Example: Fe 360 means a steel having minimum tensile strength of 360 N/mm^2 and FeE 270 means steel having yield strength of 270 N/mm^2. ... **Tb I.7/ Pg 462, DHB**

1.7.2 Steels Designated on the Basis of Chemical Composition

A. Unalloyed Steels

1. **Carbon steels:** The designation consists of the following in order.
   i. Figure indicating 100 times the average percentage of carbon content
   ii. Letter 'C'
   iii. Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

   Example: 25C means a carbon steel containing 0.20 to 0.30 per cent (0.25 per cent on average) carbon and 0.30 to 0.60 per cent (0.45 per cent rounded off to 0.5 per cent on an average) manganese. ... **Tb I.8/ Pg 463, DHB**
2. **Unalloyed free cutting steels:** The designation consists of the following in order.
   i. Figure indicating 100 times the average percentage of carbon
   ii. Symbol 'C' for tool steel
   iii. Figure indicating 10 times the average percentage of manganese
   iv. Symbol 'S' followed by the figure indicating the 100 times the average content of sulphur.

   Example: 35C10S14K indicates free cutting steel with 0.35% (average) carbon, 1% manganese and 0.14% sulphur. ... **App. I/ Pg 455, DHB**
3. **Unalloyed tool steels:** The designation consists of the following in order.
   i. Figure indicating 100 times the average percentage of carbon
   ii. Letter 'T'
   iii. Figure indicating 10 times the average percentage of manganese.

   Example: 75T5 indicates unalloyed tool steel with average 0.75% carbon and 0.5% manganese. ... **App. I/ Pg 455, DHB**

B. Alloy Steels

Alloy steel may be defined as a steel to which elements other than carbon are added in sufficient amount to produce an improvement in properties. The commonly added elements are nickel, chromium, manganese, silicon, molybdenum, cobalt, vanadium and tungsten.

1. **Low and medium alloy steels:** The term low and medium alloy steels is used for alloy steels containing less than 10% of alloying elements. The designation consists of the following in order.
   i. Figure indicating 100 times the average percentage carbon.
   ii. Chemical symbol for alloying elements each followed by the figure for its average percentage content multiplied by a factor as given below.

Element	*Multiplying factor*
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S and N	100

... **App. I/ Pg 456, DHB**

Note:
- The figure after multiplying shall be rounded off to the nearest integer.
- Symbol 'Mn' for manganese shall be included in case manganese content is equal to or greater than 1%.

- The chemical symbols and their figures shall be listed in the designation in the order of decreasing content.

Example: 40Cr4Mo2 means alloy steel having 0.4% average carbon, 1% chromium and 0.20% molybdenum. ... **App. I/ Pg 456, DHB**

2. **High alloy steels:** The term high alloy steels is used for alloy steels containing more than 10% of alloying elements. The designation consists of the following in order.
   i. Letter 'X'
   ii. Figure indicating 100 times the percentage of carbon content
   iii. Chemical symbol for alloying elements each followed by a figure for its average percentage content rounded off to the nearest integer
   iv. Chemical symbol to indicate specially added element to allow the desired properties.

   Example: X15Cr25Ni12 means high alloy steel with 0.15% carbon, 25% chromium and 12% nickel. ... **App. I/ Pg 456, DHB**

3. **Alloy tool steels**
   i. *Low and medium alloy tool steel*: The designation consists of the following in order.
      a. Letter 'T'
      b. Figure indicating 100 times the percentage of carbon content
      c. Chemical symbol for alloying elements each followed by the figure for its average percentage content rounded off to the nearest integer
      d. Chemical symbol to indicate specially added element to attain the desired properties.

      Example: X75W18Cr4V1 means low alloy tool steel having 0.75% average carbon, 18% tungsten, 4% chromium and 1% vanadium. ... **App. I/ Pg 456, DHB**

   ii. *High alloy tool steel*: The designation consists of the following in order.
      a. Letter 'XT'
      b. Figure indicating 100 times the percentage of carbon content
      c. Chemical symbol for alloying elements each followed by the figure for its average percentage content rounded off to the nearest integer
      d. Chemical symbol to indicate specially added element to attain the desired properties.

      Example: XT98W6Mo5Cr4V1 means a high alloy tool steel having 0.98% average carbon, 6% tungsten, 5% molybdenum, 4% chromium and 1% vanadium. ... **App. I/ Pg 456, DHB**

4. **Free cutting alloy steels:** The designation is same as indicate in alloy tool steels except that depending upon the percentage or S, Se, Te and Zr present, the designation shall also consist of the chemical symbol of the element present followed by the figure indicating 100 times its content.
   Example: X15Cr25Ni15S40 means alloy free cutting steel having 0.15% carbon, 25% chromium, 15% nickel and 0.40% vanadium. ... **App. I/ Pg 457, DHB**

1.7.3 Designation of Ferrous Castings ... App. I/ Pg 456, DHB

1. **On the basis of mechanical properties:** The designation used is *300.
   * refers to any of the symbols listed below
   FG = grey iron castings
   SG = spheroidal or nodular graphite iron
   CS = steel castings etc.
   300 = minimum tensile strength in N/mm^2.

2. **On the basis of chemical composition:** The designation used is *33Ni4Cr2.

* refers to any of the symbols listed in **App. I/Pg 457, DHB.**

33 = total carbon 3.3%
Ni4 = 4% nickel (average)
Cr2 = 2% chromium.

1.7.4 Designation of Pig Iron

The designation used is PG 12 Mn3P15. ... **App. I/ Pg 457, DHB**

PG = designation for pig iron
12 = indicates silicon as four times its average percentage rounded to the nearest integer without its symbol (Si)
Mn3 = indicates manganese as four times its average percentage rounded to the nearest integer
P15 = indicates phosphorous as 100 times the average percentage.

1.8 STRESS ANALYSIS (FUNDAMENTALS FROM STRENGTH OF MATERIALS)

1.8.1 Load (*F*)

Load may be defined as the combined effect of external forces acting on a body.

- The loads may be classified as:
  a. Dead/steady load
  b. Live/variable/fluctuating load
  c. Inertia load
  d. Centrifugal load.
- Apart from these, loads may also be classified as:
  a. Tensile load
  b. Compressive load
  c. Torsional/twisting load
  d. Bending load
  e. Shear load.
- Further, loads can also be either as:
  a. Point/concentrated load
  b. Distributed load.

In general, loads can be of following types.

- *Transverse loading*: It refers to the forces applied perpendicular to the longitudinal axis of a member. Transverse loading causes the member to bend and deflect from its original position, with internal tensile and compressive strains accompanying change in curvature. Transverse loading also induces shear forces that cause shear deformation of the material and increase the transverse deflection of the member.
- *Axial loading*: Here the applied forces are collinear with the longitudinal axes of the member. The forces cause the member to either stretch or shorten.
- *Torsional loading*: Twisting action caused by a pair of externally applied equal and oppositely directed couples acting on parallel planes or by a single external couple applied to a member that has one end fixed against rotation.

1.8.2 Stress

When a body is subjected to a load within the elastic limits, it develops an equal and opposite resisting force within the body. This resisting force per unit area is called stress.

$$\sigma = \pm F/A \text{ (MPa or N/mm}^2) \quad \ldots \text{(Eq. 1.1)} \ \textbf{1.1(a)/ Pg 2, DHB}$$

where F = force [(+)tensile, (–) compressive] (N)
A = cross-sectional area (mm^2)

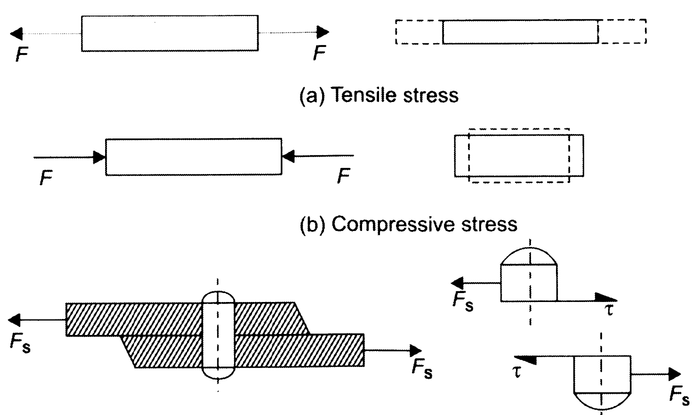
It is a property at a specific point within a body, which is a function of load, geometry, temperature, and manufacturing processing.

The stresses are of the following types.

- *Tensile stress*: It is the stress state caused by an applied load that tends to elongate the material in the axis of the applied load, in other words, the stress caused by pulling the material. Due to this the length of the member increases, while its cross-section decreases, as shown in **Fig. 1.2(a).**
  The strength of structures of equal cross-sectional area loaded in tension is independent of shape of the cross-section. Materials loaded in tension are susceptible to stress concentrations such as material defects or abrupt changes in geometry. However, materials exhibiting ductile behavior (most metals, for example) can tolerate some defects while brittle materials (such as ceramics) can fail well below their ultimate material strength.
- *Compressive stress*: It is the stress state caused by an applied load that acts to reduce the length of the material (compression member) in the axis of the applied load. In other words, the stress state caused by squeezing/pushing the material. Due to this the length of the member decreases, while its cross-section increases, as shown in **Fig. 1.2(b).**
  Compressive strength for materials is generally higher than their tensile strength. However, structures loaded in compression are subject to additional failure modes dependent on geometry, such as Euler buckling.
  In general the tensile and compressive stresses are referred to as normal stresses.
- *Shear stress* (τ): It is the stress state caused by a pair of opposing forces acting along parallel lines of action through the material, in other words, the stress caused by faces of the material sliding relative to one another.
  An example is cutting paper with scissors or stresses due to torsional loading, or a rivet subjected to tensile loading, as shown in **Fig. 1.2(c).**

$$\tau = F_s/A_s \text{ (MPa or N/mm}^2) \quad \ldots \text{(Eq. 1.2)} \ \textbf{1.1(c)/ Pg 2, DHB}$$

where F_s = shear force (N); A_s = shear area (mm^2)



(c) Shear stress

Fig. 1.2: Types of stresses

1.8.3 Strain (ε)

The deformation of a body per unit length is called strain. The deformation can be elongation/contraction.

$$\varepsilon = \delta / L \qquad \text{... (Eq. 1.3)}$$

where δ = deformation (mm), L = length (mm)

The strains are of the following types.

- *Tensile strain*: When a body is subjected to tensile stress, the strain produced in the body is referred to as tensile strain. Due to this the length of the member increases, while its cross-section decreases, as shown in **Fig. 1.2(a)**.
- *Compressive strain*: When a body is subjected to compressive stress, the strain produced in the body is referred to as compressive strain. Due to this the length of the member decreases, while its cross-section increases, as shown in **Fig. 1.2(b)**.
- *Shear strain* (γ): When a body is subjected to shear stress, the strain produced in the body is referred to as shear strain.

$$\gamma = \delta / L \qquad \text{... (Eq. 1.4)}$$

1.8.4 Hooke's Law

It states that, "when a body is loaded within the elastic limits, stress is directly proportional to strain", i.e. $\sigma \propto \varepsilon$

i.e. $\sigma = E \cdot \varepsilon$

$E = \sigma / \varepsilon$

or $\varepsilon = \sigma / E = F / AE$ ... (Eq. 1.5) **1.2(a)/ Pg 2, DHB**

$\therefore$ $\delta = FL / AE = \sigma L / E$ ... *using (Eqs 1.1 and 1.3)*

... (Eq. 1.6) **1.2(b)/ Pg 3, DHB**

where E is a proportionality constant, known as ***modulus of elasticity or Young's modulus***.

1.8.5 Shear Modulus/Modulus of Rigidity (G)

It is defined as the ratio of shear stress to shear strain

i.e. $G = \tau / \gamma$ ... (Eq. 1.7) **1.2(c)/ Pg 3, DHB**

$\therefore$ $\delta = F_s L / A_s G = \tau L / G$ ... *using (Eqs 1.2 and 1.4)* ... (Eq. 1.8)

where G = shear modulus or modulus of rigidity.

1.8.6 Poisson's Ratio

When a bar is subject to a simple tensile loading there is an increase in length of the bar in the direction of the load, but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as Poisson's ratio.

$$\mu = \frac{\text{lateral strain}}{\text{linear strain}} \qquad \text{... (Eq. 1.9)}$$

Note: Values of E, G and μ ... **Table 1.1/ Pg 11, DHB**

1.9 STRESS–STRAIN DIAGRAM

The stress–strain curve for a MS bar can be explained on the basis of tensile test performed in a Universal Testing Machine (UTM). Here the test specimen is gripped between the jaws of the machine along with a strain measuring device and load measuring device.

Referring to **Fig. 1.3**, the relation between stress and strain is initially linear (OP), where P defines the limit of proportionality. On further straining, the relation is no longer linear but is still elastic, i.e. the material regains its original shape and size upon the removal of applied load. The maximum load that can be applied without causing the material to undergo permanent deformation defines the elastic limit (PE). Point A marks the end of elastic state and initiation of plastic state, known as upper yield point (Y_U).

Upon further straining, there is a sudden drop in stress and then extension occurs at approximately constant stress, known as lower yield point (Y_L). After Y_L, stress increases with further increase in strain. This effect of the material being able to withstand greater stress despite uniform reduction in cross-sectional area is called strain or work hardening.

At S (design or working stress) the rate of work hardening is unable to keep pace with the rate of reduction in c/s area. Hence 'necking' (local strain hardening) takes place leading to fracture at B (ultimate or fracture or breaking stress).

- *Limit of proportionality* refers to the limit until which stress is directly proportional to strain.

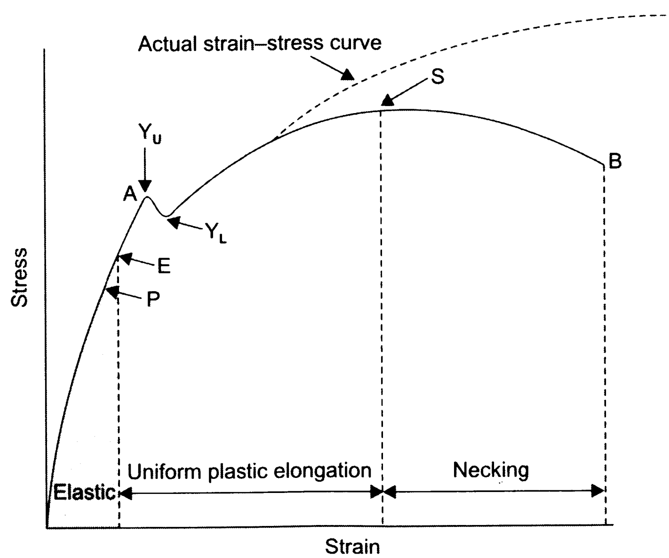


Fig. 1.3: Stress–strain curve for ductile material (MS)

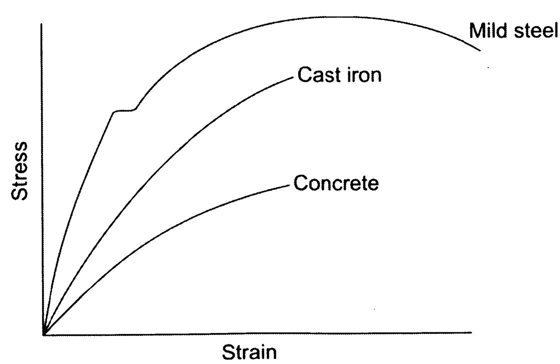


Fig. 1.4: Stress–strain diagram for various materials

- *Elastic limit*: The elastic limit is the limit beyond which the material will no longer regain its original shape when the load is removed, or it is the maximum stress that may be developed such that there is no permanent or residual deformation when the load is entirely removed.
- *Yield point*: Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.
- *Upper yield point* (Y_U): It is the stress at which the extension increases without further increase in load.
- *Lower yield point* (Y_L): Here stress remains constant while the strain increases for some time.
- *Ultimate strength* (S): The maximum ordinate in the stress–strain diagram is the ultimate strength or tensile strength.
- *Rupture strength* (B): Rupture strength is the strength of the material at rupture. This is also known as the breaking strength.
- *Modulus of resilience*: Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from O to P. This may be calculated as the area under the stress–strain curve from the origin O up to the elastic limit E (the shaded area in Fig. 1.3).
  The resilience of the material is its ability to absorb energy without creating a permanent distortion.
- *Modulus of toughness*: Modulus of toughness is the work done on a unit volume of material as the force is gradually increased from O to B. This may be calculated as the area under the entire stress–strain curve (from O to B). The toughness of a material is its ability to absorb energy without causing it to break.

Note:

- Properties of typical cast irons. ... **Table I.17/ Pg 472, DHB**
- Properties of typical carbon and alloy steels. ... **Table i.18/ Pg 473, DHB**

1.10 MATERIAL SUBJECTED TO COMBINED DIRECT AND SHEAR STRESS (GENERAL 2D STRESS SYSTEM)

Figure 1.5(a) represents a stress element subjected to direct stresses σ_x, σ_y and shear stress τ_{xy}. Let this element be cut by a plane *AB*, whose normal makes an angle ϕ with Y-axis as shown in **Fig. 1.5(b)**. On this inclined plane, we have:

- Normal stress, σ_n acting at right angles to the plane *AB*
- Shear stress, τ acting along or parallel to *AB*

The magnitudes of these stresses are

- Normal stress, $\sigma_n = \left(\dfrac{\sigma_x + \sigma_y}{2}\right) + \left(\dfrac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi + \tau_{xy}\sin 2\phi$

  ... (Eq. 1.10) **1.8(a)/ Pg 5, DHB**

- Shear stress, τ or $\tau_n = \left(\dfrac{\sigma_x - \sigma_y}{2}\right)\sin 2\phi - \tau_{xy}\cos 2\phi$ ... (Eq. 1.11) **1.8(b)/ Pg 5, DHB**
- The resultant stress is calculated as $\sigma = \sqrt{\sigma_n^2 + \tau^2}$ (τ or τ_n)

  ... (Eq. 1.12) **1.7(g)/ Pg 4, DHB**

In order to find the principal planes, the maximum and minimum values of σ_n must be obtained by equating τ to zero. Thus, we have

$$\tan 2\phi_1 = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

... (Eq. 1.13) **1.8(e)/ Pg 5, DHB**

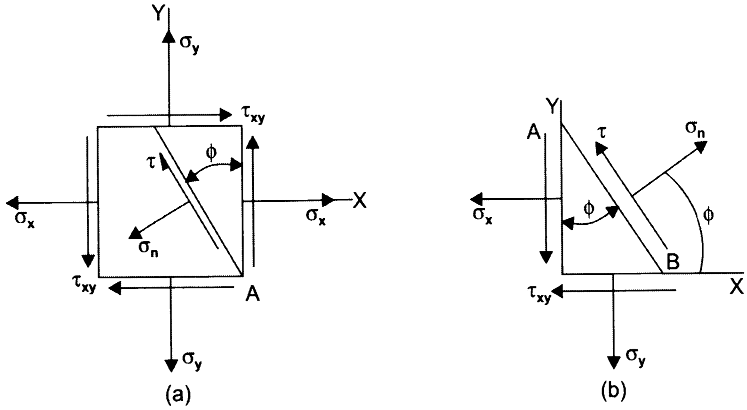


Fig. 1.5: The general condition of two dimensional stress **[Fig. 1.4(a) Plane stress system/Pg 5, DHB]**

Equation 1.13 can be represented as shown in **Fig. 1.6(a).** From this equation we have two values of $2\phi_1$ which differ by 180° [i.e. ϕ_1 and ϕ_2 differ by 90°].

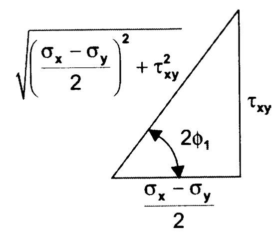


Fig. 1.6a: Direction of principal stress **[Fig. 1.4(b)/Pg 5, DHB]**

- The values of $2\phi_1$ obtained from (Eq. 1.13) are 180° apart. Equation (1.13) is satisfied at two possible values of ϕ_1. Substituting this value of $2\phi_1$ in (Eq. 1.10) we get two principal stresses, represented by σ_1 and σ_2, which is given as:

Maximum principal stress,

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (Eq. 1.14) } \mathbf{1.8(c)/\ Pg\ 5,\ DHB}$$

Minimum principal stress,

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (Eq. 1.15) } \mathbf{1.8(d)/\ Pg\ 5,\ DHB}$$

- Further there are two values of $2\phi_1$ at which the shear stress are maximum. For maximum value of τ, differentiating (Eq. 1.11) with respect to ϕ and equating to zero, we have

$$\tan 2\phi_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \quad \text{... (Eq. 1.16) } \mathbf{1.8(g)/\ Pg\ 5,\ DHB}$$

Equation 1.16 can be represented as shown in **Fig. 1.6(b).** From this equation we have two values of $2\phi_s$ which differ by 180° [i.e. ϕ_{s1} and ϕ_{s2} differ by 90°].

- The values of $2\phi_s$ obtained from (Eq. 1.16) are 180° apart and the maximum shear stress occurs on planes that are 90° apart. The maximum shear stress is found as:

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (Eq. 1.17) } \mathbf{1.8(f)/\ Pg\ 5,\ DHB}$$

In terms of principal stresses,

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \qquad \ldots \text{(Eq. 1.18)} \ \mathbf{1.8(f)/\ Pg\ 5,\ DHB}$$

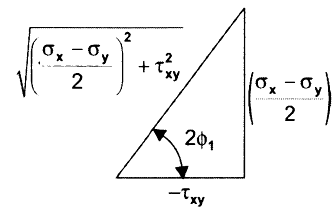


Fig. 1.6b: Direction of shear stress **[Fig. 1.4(c)/Pg 5, DHB]**

Note:

- Direction of principal stresses:

$$\phi_1 = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \text{ and } \phi_2 = \phi_1 + 90°$$

- Direction of shear stresses:
  $\phi_{s\,max} = \phi_{s1} = \phi_1 + 45°$ and $\phi_{s\,min} = \phi_{s2} = \phi_1 + 135° = \phi_{s1} + 90°$

1.11 BIAXIAL/ TWO DIMENSIONAL STRESS

When a component is subjected to different forces such that the stresses produced act only on two planes perpendicular to each other then the state of stress is referred to as ***biaxial stress***, as shown in **Fig. 1.7.**

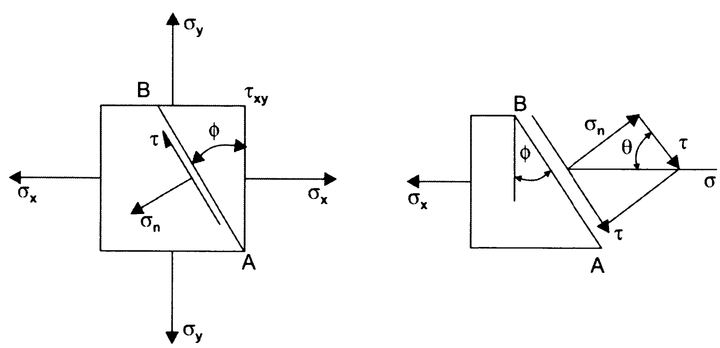


Fig. 1.7: Biaxial stress system **[Fig. 1.3(b)/Pg 4, DHB]**

Figure 1.7 represents a stress element subjected to direct stresses σ_x and σ_y. Let this element be cut by a plane *AB*, whose normal makes an angle ϕ with Y-axis as shown. On this inclined plane, we have:

- Normal stress, σ_n acting at right angles to the plane *AB*
- Shear stress, τ acting along or parallel to '*AB*'

The magnitudes of these stresses are

- Normal stress, $$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi$$

... (Eq. 1.19) **1.7(a)/ Pg 4, DHB**

- Shear stress, $$\tau \text{ or } \tau_n = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin^2\phi$$ ... (Eq. 1.20) **1.7(b)/ Pg 4, DHB**

- The resultant stress is calculated as $\sigma = \sqrt{\sigma_n^2 + \tau^2}$ ... (Eq. 1.21) **1.7(g)/ Pg 4, DHB**

- Direction of resultant stress or angle of obliquity, $\tan\theta = \left(\frac{\sigma_n}{\tau}\right)$

... (Eq. 1.22) **1.7(h)/ Pg 4, DHB**

Note:

- Referring to **Fig. 1.3(b)/Pg 4, DHB,** equations for biaxial stress system are available under **Eq. 1.7(a) to 1.7(h)/ Pg 4, DHB.**
- Referring to **Fig. 1.3(a)/Pg 4, DHB,** equations for uniaxial stress system are available under **Eq. 1.6(a) to 1.6(f)/Pg 4, DHB.**

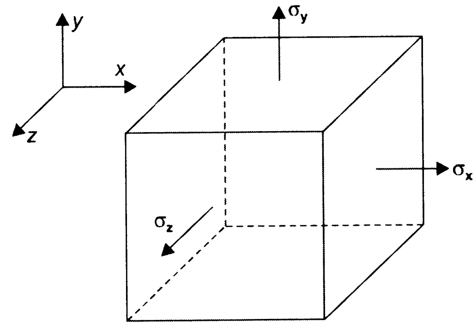


Fig. 1.8: Triaxial stress system **[Fig. 1.7(a)/ Pg 7, DHB]**

1.12 TRIAXIAL STRESS/THREE DIMENSIONAL STRESS

A state of **triaxial stress** exists in an element if it is subjected to normal stresses σ_x, σ_y and σ_z in three mutually perpendicular directions and there are no shear stresses on the faces of the element as shown in Fig. 1.8. Since there are no shear stresses on the x, y and z faces, the stresses σ_x, σ_y and σ_z are the principal stresses (σ_1, σ_2 and σ_3) in the material, i.e. the principal normal stresses are σ_1, σ_2 and σ_3 where $\sigma_1 \geq \sigma_2 \geq \sigma_3$, then τ_{max} is largest of

$$\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_1 - \sigma_3}{2} \qquad \text{... (Eq. 1.23)}$$

1.13 STRESS TENSOR/COMPONENTS OF STRESS

Consider a rectangular parallelepiped (cubical element) as shown in **Fig. 1.9.** The state of stress at any point of a continuous body is determined entirely by the components of stress in three mutually perpendicular planes which pass through the chosen point. These planes are usually taken perpendicular to the orthogonal coordinate system.

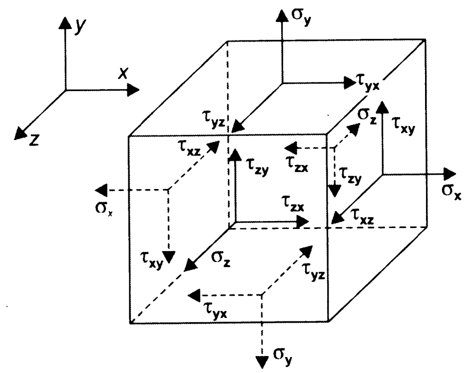


Fig. 1.9: Components of stress

To define the stresses acting on six faces of the cubical element, we need three normal stresses: σ_x, σ_y and σ_z and six shear stresses: τ_{xy}, τ_{yx}, τ_{yz}, τ_{zy}, τ_{zx} and τ_{xz}. Thus the state of stress at a point using three mutually perpendicular (orthogonal) planes can be described by the nine distinct values of stress represented in the matrix form shown below.

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \text{... (Eq. 1.24)}$$

To ensure the static equilibrium, the following equations must hold.

$$\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \text{ and } \tau_{zx} = \tau_{xz} \qquad \text{... (Eq. 1.25)}$$

$$\therefore \text{ Equation (1.24) yields } \tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \text{... (Eq. 1.26)}$$

These components together form a ***'stress tensor' or 'stress matrix'***. A tensor matrix describes the property of a system which is invariant with respect to coordinate system.

Notation of stress: The normal stress is represented by σ_n and the tangential (shear) stress by τ. Subscripts are used to indicate the direction of the plane on which the stress is acting. For example, σ_x indicates that the stress is acting on a plane normal to the X-axis.

The shear stress is further resolved into two components parallel to the coordinate axis. Two subscripts are used in this case:

- The first indicating the direction of normal to the plane (face) and
- The second indicates the direction of component of stress as shown below:

$$\tau_{mn}$$

Face Direction

This shows that the first is the plane in which the stress acts and the second is the direction in which the stress acts.

For example, τ_{yz} is the shear stress in the plane perpendicular to Y-axis in the Z-direction. Each component represents a magnitude for that particular plane and direction.

1.14 BIAXIAL DEFORMATION/STRAIN

For a 2D state of stress, refer **Fig. 1.7 [Fig. 1.3(b)/ Pg 4, DHB]**

- Poisson's ratio $\mu = \dfrac{-\varepsilon_y}{\varepsilon_x} = \dfrac{-\varepsilon_z}{\varepsilon_x}$ ... (Eq. 1.27) **1.11(a)/ Pg 7, DHB**
- Resultant strain in X-direction is $\varepsilon_x = \dfrac{\sigma_x}{E} - \mu\dfrac{\sigma_y}{E}$ ... (Eq. 1.28) **1.11(b)/ Pg 7, DHB**
- Resultant strain in Y-direction is $\varepsilon_y = \dfrac{\sigma_y}{E} - \mu\dfrac{\sigma_x}{E}$ ... (Eq. 1.29) **1.11(c)/ Pg 7, DHB**

Thus, the principal stresses in terms of principal strains are calculated as below.

- Normal stress in X-direction, $\sigma_x = \dfrac{(\varepsilon_x + \mu\varepsilon_y)E}{1-\mu^2}$ ... (Eq. 1.30) **1.11(d)/ Pg 7, DHB**
- Normal stress in Y-direction, $\sigma_y = \dfrac{(\varepsilon_y + \mu\varepsilon_x)E}{1-\mu^2}$ ... (Eq. 1.31) **1.11(e)/ Pg 7, DHB**

1.15 TRIAXIAL DEFORMATION/STRAIN

On similar lines, for a 3D state of stress, **Fig. 1.9 [Fig. 1.7(b)/ Pg 7, DHB]**

- Resultant strain in X-direction is

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)]$$ ... (Eq. 1.32) **1.12(a)/ Pg 7, DHB**

- Resultant strain in Y-direction is

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_z + \sigma_x)]$$ ... (Eq. 1.33) **1.12(b)/ Pg 7, DHB**

- Resultant strain in Z-direction is

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)] \qquad \text{... (Eq. 1.34) } \mathbf{1.12(c)/\ Pg\ 7,\ DHB}$$

1. **Identify the following engineering materials giving specifications:**
   i. **FG350** ii. **FeE300** iii. **C35Mn75** iv. **X20Cr18Ni2**

   VTU – June 12 – 04 Marks

   Solution:
   i. FG350: Grey cast iron with ultimate strength of 350 N/mm^2.
   ... **Tb I.4/ Pg 461, DHB**
   ii. FeE300: Plain carbon steel having yield strength of 360 N/mm^2.
   ... **Tb I.7/ Pg 462, DHB**
   iii. C35Mn75: Free cutting steel with 0.35% (average) carbon, 0.75% manganese.
   ... **Tb I.8/ Pg 463, DHB**
   iv. X20Cr18Ni2: High alloy steel with 0.20% carbon, 18% chromium and 2% nickel.
   ... **Pg 456, DHB**

2. **The principal stresses at a point in a strained material are σ_x and σ_y. Show that the resultant stress σ_r on the plane carrying the maximum shear stress is**

$$\sigma_r = \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{2}}$$

Solution:
We know that the resultant stress,

$$\sigma_r = \sqrt{\sigma_n^2 + \tau^2} \qquad \text{... Eq. (i) } \mathbf{1.7(g)/\ Pg\ 4,\ DHB}$$

But normal stress, $$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi + \tau_{xy}\sin 2\phi$$

... **1.8(a)/ Pg 5, DHB**

i.e. $$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi \text{ (since } \tau_{xy} = 0)$$

... Eq. (ii) **1.7(a)/ Pg 4, DHB**

and shear stress, $$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\phi - \tau_{xy}\cos 2\phi \qquad \text{... } \mathbf{1.8(b)/\ Pg\ 5,\ DHB}$$

i.e. $$\tau \text{ or } \tau_n = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\phi$$

... Eq. (iii) **1.7(b)/ Pg 4, DHB**

For maximum shear stress, $\phi = 45°$

$\therefore$ Eq. (i) yields

$$\sigma_r = \sqrt{\left[\left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos(2\times 45)°\right]^2 + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\times 45)°\right]}$$

$$= \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

i.e. $$\sigma_r^2 = \frac{1}{4}(\sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y) + \frac{1}{4}(\sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y)$$

$$= \frac{1}{4}(2\sigma_x^2 + 2\sigma_y^2)$$

$$= \frac{1}{2}(\sigma_x^2 + \sigma_y^2)$$

$\therefore$ $$\sigma_r = \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{2}}$$

3. **At a point in a 2D stress system, the normal stresses on two mutually perpendicular planes are σ_x and σ_y (both alike) and the shear stress is τ_{xy}. If $\tau_{xy}^2 = \sigma_x \cdot \sigma_y$, show that one of the principal stresses is zero.**

Solution:

We know that $$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$ **... 1.8(c)&(d)/ Pg 5, DHB**

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\frac{\sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y}{4} + \sigma_x\sigma_y}$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\frac{\sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y + 4\sigma_x\sigma_y}{4}}$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + 2\sigma_x\sigma_y}{4}}$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2}$$

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

$\therefore$ $$\sigma_1 = (\sigma_x + \sigma_y) \text{ and } \sigma_2 = 0$$

4. **A rectangular bar of section 50 mm × 25 mm is subjected to a tensile load of 25 kN. Determine the values of normal and shear stresses on a plane 30° with the vertical. Also calculate the magnitude and direction of the maximum shear stress.**

VTU – Dec.08/ Jan.2009 – 08 Marks

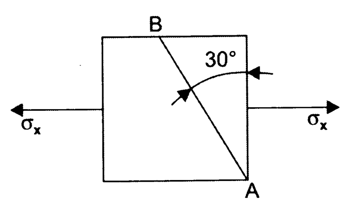


Fig. 1.10: Problem 4

Solution: $A = 50 \times 25 = 1250$ mm^2, $F = 25$ kN, $\sigma_n = ?$, $\tau = ?$, $\phi = 30°$.

- We know that $\sigma_n = \sigma_x \cos^2\phi$ **... 1.6(a)/ Pg 4, DHB**

But $$\sigma_x = \frac{F}{A} = \frac{25000}{1250} = 20 \text{ MPa}$$ **... 1.1(a)/ Pg 2, DHB**

$$\sigma_n = 20 \times \cos^2(30) = 15 \text{ MPa}$$

- Also $$\tau = \frac{\sigma_x}{2}\sin 2\phi = \frac{20}{2}\sin(2 \times 30) = 8.66 \text{ MPa}$$

... 1.6(b)/ Pg 4, DHB

- Maximum shear stress

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{20}{2} = 10 \text{ MPa}$$ **... 1.6(e)/ Pg 4, DHB**

- Direction of maximum shear stress

$$\phi_s = 45°$$

5. For the system shown in Fig. 1.11, determine the normal and tangential stress intensities. Also find the direction of resultant stress.

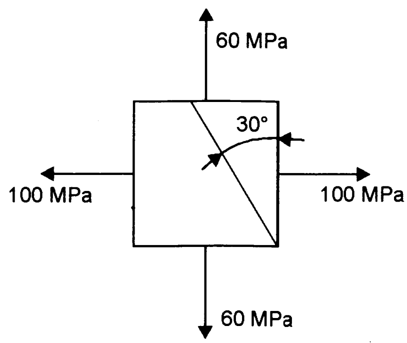


Fig. 1.11: Problem 5

Solution: $\sigma_x = 100$ MPa, $\sigma_y = 60$ MPa, $\phi = 30°$, $\sigma_n = ?$, $\tau = ?$, $\theta = ?$

- Normal stress $$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi$$ **... 1.7(a)/ Pg 4, DHB**

$$= \left(\frac{100 + 60}{2}\right) + \left(\frac{100 - 60}{2}\right)\cos(2 \times 30)$$

$$\sigma_n = 90 \text{ MPa}$$

- Shear stress $$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\phi$$ **... 1.7(b)/ Pg 4, DHB**

$$= \left(\frac{100 - 60}{2}\right)\sin(2 \times 30)$$

$$\tau = 17.32 \text{ MPa}$$

- Direction of resultant stress of angle of obliquity,

$$\tan\theta = \left(\frac{\sigma_n}{\tau}\right) = \left(\frac{90}{17.32}\right)$$ **... 1.7(h)/ Pg 4, DHB**

$$\theta = 79.11°$$

6. A point in a structural member is subjected to plane state of stress as shown in Fig. 1.12. Determine the following:

i. Normal and tangential stress intensities at an angle of θ = 45°

ii. **Principal stresses σ_1 and σ_2 and their directions**

iii. **Maximum shear stress and its plane.**

VTU – June 2012 – 10 Marks; Dec.09/ Jan.2010 – 10 Marks; (Similar) June/ July 2011 – 14 Marks; (similar) June/ July 2015 – 10 Marks

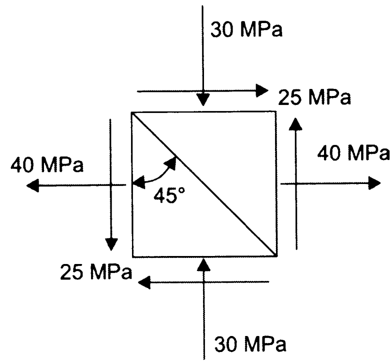


Fig. 1.12: Problem 6

Solution: $\sigma_x = 40$ MPa, $\sigma_y = -30$ MPa, $\tau_{xy} = 25$ MPa, $\phi_n = 45°$.

i. **Normal and tangential stress intensities:** We know that normal stress

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi + \tau_{xy}\sin 2\phi$$

... 1.8(a)/ Pg 5, DHB

$$= \left(\frac{40-30}{2}\right) + \left[\left(\frac{40+30}{2}\right)\cos(2\times 45)\right] + 25\sin(2\times 45)$$

$$\sigma_n = 30 \text{ MPa}$$

Also tangential stress,

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\phi - \tau_{xy}\cos 2\phi \qquad \text{... 1.8(b)/ Pg 5, DHB}$$

$$= \left[\left(\frac{40+30}{2}\right)\sin(2\times 45)\right] - 25\cos(2\times 45)$$

$$\tau = 35 \text{ MPa}$$

ii. **Principal stresses and their directions:**

We know that $\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$ **... 1.8(c)&(d)/ Pg 5, DHB**

$$= \left(\frac{40-30}{2}\right) \pm \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

$\therefore$ $\sigma_{1,2} = 48.01, -38.01$ MPa

i.e. $\sigma_1 = 48.01$ MPa

and $\sigma_2 = -38.01$ MPa

Directions: We know that

$$\tan 2\phi_1 = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \quad \ldots \textbf{1.8(e)/ Pg 5, DHB}$$

$$\tan 2\phi_1 = \left(\frac{2 \times 25}{40 - (-30)}\right) = 0.714$$

$$2\phi_1 = 35.54°$$

$$\therefore \quad \phi_1 = 17.77°$$

$$\phi_2 = \phi_1 + 90° = 17.77° + 90° = 107.77°$$

iii. **Maximum shear stress:** We know that

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \ldots \textbf{1.8(f)/ Pg 5, DHB}$$

$$= \pm\sqrt{\left(\frac{40 + 30}{2}\right)^2 + 25^2}$$

$$\tau_{max} = \pm 43.01 \text{ MPa}$$

Directions: We know that

$$\tan 2\phi_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \quad \ldots \textbf{1.8(g)/ Pg 5, DHB}$$

$$\tan 2\phi_s = -\left(\frac{40 - (-30)}{2 \times 25}\right) = -1.4$$

$$2\phi_s = -54.46°$$

$$\therefore \quad \phi_{s1} = -27.23°$$

and $\phi_{s2} = \phi_1 + 90° = -27.23° + 90° = 62.77°$

or Direction of maximum shear stress,

$$\phi_{s1} = \phi_1 + 45° = 17.77 + 45° = 62.77°$$

Direction of minimum shear stress

$$\phi_{s2} = \phi_1 + 135° = 17.77 + 135° = 152.77° \quad [\text{or } \phi_{s1} + 90°]$$

7. **A machine component is subjected to stresses as shown in Fig. 1.13. Determine:**

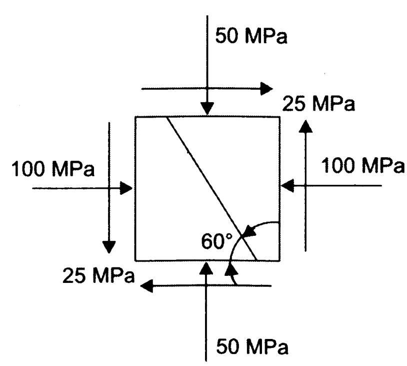


Fig. 1.13: Problem 7

a. Normal, shear and resultant stresses
b. Principal stresses and their directions

Solution: $\sigma_x = -100$ MPa, $\sigma_y = -50$ MPa, $\tau_{xy} = 25$ MPa, $\phi' = 60°$ (with horizontal), $\phi' = 90° - 30°$ (with vertical).

a. **Normal, shear and resultant stresses:** We know that normal stress

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi + \tau_{xy}\sin 2\phi$$

... 1.8(a)/ Pg 5, DHB

$$= \left(\frac{-100 - 50}{2}\right) + \left[\left(\frac{-100 + 50}{2}\right)\cos(2 \times 30)\right] + 25\sin(2 \times 30)$$

$$\sigma_n = -65.85 \text{ MPa}$$

Also shear stress,

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\phi - \tau_{xy}\cos 2\phi$$

... 1.8(b)/ Pg 5, DHB

$$= \left[\left(\frac{-100 + 50}{2}\right)\sin(2 \times 30)\right] - 25\cos(2 \times 30)$$

$$\tau = -34.15 \text{ MPa}$$

and resultant stress,

$$\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$$

... 1.7(g)/ Pg 4, DHB

$$= \sqrt{(-65.85)^2 + (-34.15)^2}$$

$$\sigma_r = 74.18 \text{ MPa}$$

b. **Principal stresses and their directions:** We know that

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

... 1.8(c)&(d)/ Pg 5, DHB

$$= \frac{-100 - 50}{2} \pm \sqrt{\left(\frac{-100 + 50}{2}\right)^2 + 25^2}$$

$\therefore$ $\quad \sigma_{1,2} = -39.64, -110.36$ MPa

i.e. $\quad \sigma_1 = -39.64$ MPa

and $\quad \sigma_2 = -110.36$ MPa

Directions: We know that

$$\tan 2\phi_1 = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

... 1.8(e)/ Pg 5, DHB

$$\tan 2\phi_1 = \left(\frac{2 \times 25}{-100 - (-50)}\right) = -1$$

$$2\phi_1 = -45°$$

$\therefore$ $\quad \phi_1 = -22.5°$

and $\quad \phi_2 = \phi_1 + 90° = -22.5° + 90° = 67.5°$

8. A point in a structural member is subjected to plane stress as shown in Fig. 1.14. Determine (a) the principal stresses and (b) their directions.

VTU – Dec.13/ Jan.2014 – 08 Marks

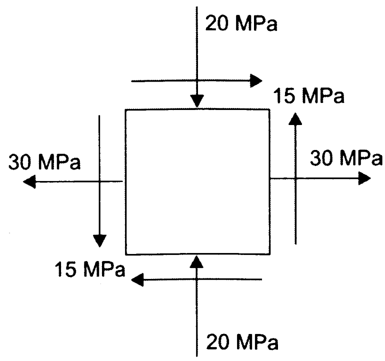


Fig. 1.14: Problem 8

Solution: $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa, $\tau_{xy} = 15$ MPa.

a. **Principal stresses:** We know that

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

... **1.8(c)&(d)/ Pg 5, DHB**

$$= \left(\frac{30 - 20}{2}\right) \pm \sqrt{\left(\frac{30 + 20}{2}\right)^2 + 15^2}$$

$\therefore$ $\sigma_{1,2} = 34.15$ MPa, -24.15 MPa

i.e. $\sigma_1 = 34.15$ MPa

and $\sigma_2 = -24.15$ MPa

b. **Principal stresses directions:** We know that

$$\tan 2\phi_1 = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

... **1.8(e)/ Pg 5, DHB**

$$\tan 2\phi_1 = \left(\frac{2 \times 15}{30 - (20)}\right) = 0.6$$

$\therefore$ $2\phi_1 = 30.96°$

i.e. $\phi_1 = 15.48°$

and $\phi_2 = \phi_1 + 90° = 15.48° + 90° = 105.48°$

9. For the stress element shown in Fig. 1.15, find (a) the principal stresses and (b) directions.

VTU – June/ July 2014 – 06 Marks

Solution: $\sigma_x = 180$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa.

a. **Principal stresses:** We know that

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

... **1.8(c)&(d)/ Pg 5, DHB**

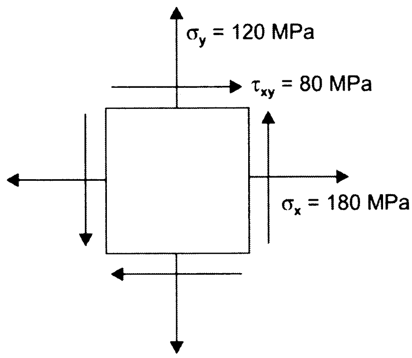


Fig. 1.15: Problem 9

$$= \left(\frac{180+120}{2}\right) \pm \sqrt{\left(\frac{180-120}{2}\right)^2 + 80^2}$$

$\therefore$ $\sigma_{1,2} = 235.44$ MPa, 64.56 MPa

i.e. $\sigma_1 = 235.44$ MPa

and $\sigma_2 = 64.56$ MPa

b. **Principal stresses directions:** We know that

$$\tan 2\phi_1 = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \quad \text{... 1.8(e)/ Pg 5, DHB}$$

$$\tan 2\phi_1 = \left(\frac{2\times 80}{180-120}\right) = 2.67$$

$\therefore$ $2\phi_1 = 69.44°$

i.e. $\phi_1 = 34.72°$

and $\phi_2 = \phi_1 + 90° = 34.72° + 90° = 124.72°$

10. For the system in Fig. 1.16, find:

a. Principal stresses and their directions

b. Maximum shear stress and their planes.

Solution: $\sigma_x = 70$ MPa, $\sigma_y = -35$ MPa, $\tau_{xy} = -17.5$ MPa.

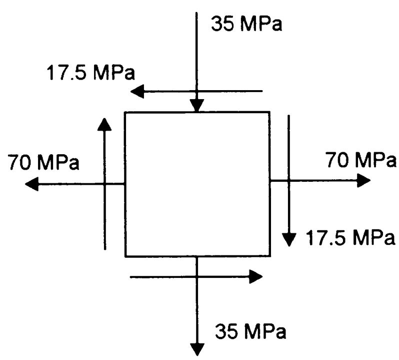


Fig. 1.16: Problem 10

a. **Principal stresses:** We know that

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... 1.8(c)\&(d)/ Pg 5, DHB}$$

$$= \left(\frac{70 - 35}{2}\right) \pm \sqrt{\left(\frac{70 + 35}{2}\right)^2 + (-17.5)^2}$$

$\therefore$ $\sigma_{1,2} = 72.84, -37.84$ MPa

i.e. $\sigma_1 = 72.84$ MPa

and $\sigma_2 = -37.84$ MPa

Principal stresses directions: We know that

$$\tan 2\phi_1 = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \quad \text{... 1.8(e)/ Pg 5, DHB}$$

$$\tan 2\phi_1 = \left(\frac{2 \times (-17.5)}{70 - (-35)}\right) = -0.334$$

$2\phi_1 = -18.43°$

$\therefore$ $\phi_1 = -9.22°$

$\phi_2 = \phi_1 + 90° = -9.22° + 90° = 80.78°$

b. **Maximum shear stress:** We know that

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... 1.8(f)/ Pg 5, DHB}$$

$$= \pm\sqrt{\left(\frac{70 + 35}{2}\right)^2 + (-17.5)^2}$$

$\tau_{max} = \pm 55.34$ MPa

or $\tau_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{72.84 - (-37.84)}{2} = 55.34$ MPa

Shear stress directions: We know that

$$\tan 2\phi_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \quad \text{... 1.8(g)/ Pg 5, DHB}$$

$$\tan 2\phi_s = -\left(\frac{70 - (-35)}{2 \times (-17.5)}\right) = 3$$

$2\phi_s = 71.57°$

$\therefore$ $\phi_s = \phi_{s1} = 35.78°$

and $\phi_{s2} = \phi_{s1} + 90° = 35.78° + 90° = 125.78°$

or direction of maximum shear stress,

$\phi_{s1} = \phi_1 + 45° = -9.22° + 45° = 35.78°$

direction of minimum shear stress

$\phi_{s2} = \phi_1 + 135° = -9.22° + 135° = 125.78°$ [or $\phi_{s1} + 90°$]

11. The state of stress at a point in a structural member is shown in Fig. 1.17. The tensile principal stress is known to be 84 N/mm^2. Determine:

a. The shear stress τ_{xy}.

b. The maximum shear stress at the point and orientation of its plane

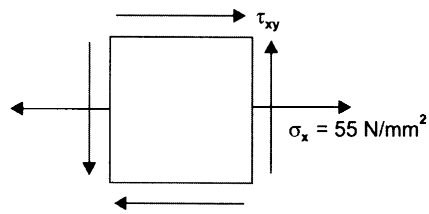


Fig. 1.17: Problem 11

Solution: $\sigma_x = 55\ \text{N/mm}^2$, $\sigma_1 = 84\ \text{N/mm}^2$ (i) $\tau_{max} = ?, 2\phi_s = ?$ (ii) $\tau_{xy} = ?$

i. **Shear stress τ_{xy}:** We know that

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{... 1.8(c)/ Pg 5, DHB}$$

$$84 = \left(\frac{55+0}{2}\right) + \sqrt{\left(\frac{55+0}{2}\right)^2 + \tau_{xy}^2}$$

$$\therefore \quad \tau_{xy} = 49.36 \text{ MPa}$$

ii. **Maximum shear stress at the point and orientation of its plane:** We know that

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \text{... 1.8(f)/ Pg 5, DHB}$$

$$= \pm\sqrt{\left(\frac{55+0}{2}\right)^2 + (49.36)^2}$$

$$\tau_{max} = \pm 56.50 \text{ MPa}$$

Shear stress directions: We know that

$$\tan 2\phi_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \qquad \text{... 1.8(g)/ Pg 5, DHB}$$

$$\tan 2\phi_s = -\left(\frac{55-0}{2 \times 49.36}\right) = -0.56$$

$$2\phi_s = -29.13^\circ$$

$$\therefore \quad \phi_{s1} = -14.56^\circ$$

and $\quad \phi_{s2} = \phi_{s1} + 90^\circ = -14.56^\circ + 90^\circ = 75.44^\circ$

VTU QUESTION PAPERS

Mar. 2001 (ME5T2)

1. Briefly discuss factors influencing the selection of suitable material for machine element. **(05 Marks)**

July/Aug. 2004 (AU46)

2. a. Draw the stress–strain diagram and name the salient points for the following materials:
   i. C60 steel **(03 Marks)**
   ii. Cast iron FG200 **(02 Marks)**
   b. On a rectangular stress element show the triaxial stress components and the corresponding stress tensor. **(05 Marks)**

Jan/Feb. 2005 (AU46)

3. a. List the three factors which govern the selection of a material for a machine component. **(03 Marks)**
   b. Define standardization. State the standards used in machine design. **(03 Marks)**

July/ Aug. 2005 (AU46)

4. Discuss factors to be considered for selection of an appropriate material for a machine element in the design process. **(08 Marks)**

Jan/Feb. 2006 (AU46)

5. a. Draw stress–strain diagram for a ductile material subjected to tension. Explain the significance of salient points. **(06 Marks)**
   b. Explain principal planes and principal stresses. **(04 Marks)**

July 2006 (AU46)

6. Draw the stress–strain diagrams for the ductile material and a brittle material and show the salient points on them. **(06 Marks)**

Dec. 07/Jan. 2008 (AU46)

7. Explain the factors which govern the selection of material for a given machine component. **(05 Marks)**

Dec. 08/Jan. 2009 (06ME-AU52)

8. a. Sketch and explain biaxial and triaxial stresses, stress tensor, and principal stresses. **(08 Marks)**
   b. A rectangular bar of section 50 mm × 25 mm is subjected to a tensile load of 25 kN. Determine the values of normal and shear stresses on a plane 30° with the vertical. Also calculate the magnitude and direction of the maximum shear stress. **(08 Marks)**
   c. Briefly explain the design codes and standards. **(04 Marks)**

June/July 2009 (AU46)

9. Define standardization. State the standards used in machine design. **(03 Marks)**

June/July 2009 (06ME52)

10. Write brief note on general procedure used in design. **(05 Marks)**

Dec. 2009/ Jan. 2010 (AU46)

11. Draw stress–strain diagram for ductile material subjected to tension. Explain the significance of salient points. **(04 Marks)**

Dec. 2009/Jan. 2010 (06ME52)

12. A point in a structural member subjected to plane stress is shown in **Fig. U1.1.** Determine the following.
    i. Normal and tangential stress intensities on a plane inclined at 45°.
    ii. Principal stresses and their directions.
    iii. Maximum shear stress and the direction of plane on which they occur.

(10 Marks)

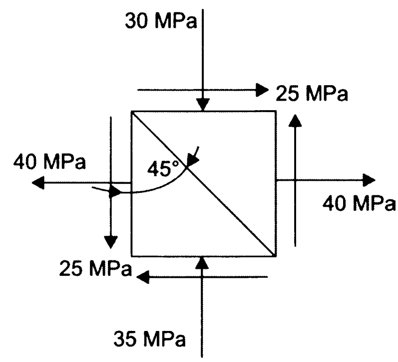


Fig. U1.1

May/June 2010 (06ME52)

13. a. Sketch and explain biaxial and triaxial stresses, shear tensor and principal stresses. **(06 Marks)**
    b. The state of stress at a point in a structural member is as shown in **Fig. U1.2**. The tensile principal stress is known to be 84 N/mm^2. Determine:
       i. The maximum shear stress at the point and orientation of its plane.
       ii. The shearing stress τ_{xy}. **(10 Marks)**
    c. Briefly discuss the factors influencing the selection of suitable material for machine element. **(04 Marks)**

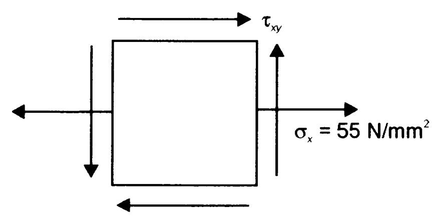


Fig. U1.2

Dec. 2010 (AU46)

14. a. Classify the standards with examples and list the purposes of standardization. **(05 Marks)**
    b. Discuss the factors which govern the selection of a material in the design of machine elements. **(05 Marks)**

Dec. 2010 (06ME52)

15. Sketch and explain biaxial, triaxial and principal stresses. **(08 Marks)**

June/ July 2011 (06ME52)

16. a. Draw the stress–strain diagram for mild steel subjected to tension. Explain the significance of salient points. **(06 Marks)**
    b. A point in a structural member subjected to plane stress is shown in **Fig. U1.3.** Determine the following.
       i. Normal and tangential stress intensities on plane MN inclined at 45°.
       ii. Principal stresses and their direction.
       iii. Maximum shear stress and the direction of the planes on which it occurs.

(14 Marks)

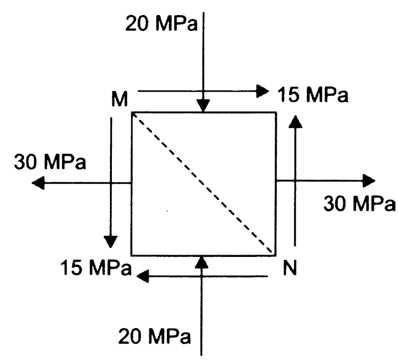


Fig. U1.3

Dec. 2011 (06ME52)

17. a. Briefly explain the important mechanical properties of metals. **(06 Marks)**
    b. Define standards and codes. **(04 Marks)**

June 2012 (06ME52-06AU52)

18. a. Identify the following engineering materials giving specifications.
    i. FG350
    ii. FeE300
    iii. C35Mn75
    iv. X20Cr18Ni2 **(04 Marks)**
    b. A point in a structural member is subjected to plane state of stress as shown in **Fig. U1.4**. Determine the following.
    i. Normal and tangential stress intensities at an angle of $\theta = 45°$.
    ii. Principle stresses σ_1 and σ_2 and their directions.
    iii. Maximum shear stress and its plane. **(10 Marks)**

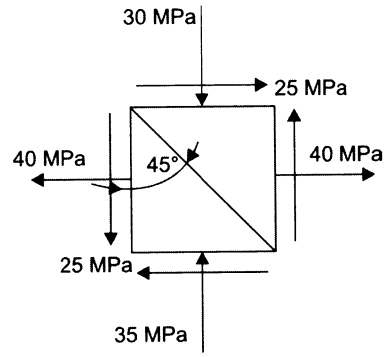


Fig. U1.4

Dec. 2012 (10ME52)

19. a. What is mechanical engineering design? Explain. **(03 Marks)**
    b. Explain the importance of standards in design. Give examples. **(03 Marks)**

Dec. 2012 (06ME52)

20. a. Draw the stress–strain curve for mild steel and cast iron. Name the salient points. **(06 Marks)**
    b. What are the important mechanical properties of metals? Explain each of them briefly. **(10 Marks)**
    c. What is standardization? What are the advantages of standardization? **(04 Marks)**

June/July 2013 (06ME52)

21. a. Briefly explain the following.
    i. Design considerations **(04 Marks)**
    ii. Design codes and standards **(02 Marks)**
    iii. Stress tensor **(02 Marks)**

June/July 2013 (10ME52)

22. a. What are the requirements of machine elements? Explain briefly. **(05 Marks)**
    b. What are the factors to be considered for selection of material for a machine component? **(05 Marks)**

Dec. 2013/Jan. 2014 (06ME52)

23. a. Write a brief note on general procedure used in design. **(05 Marks)**
    b. Discuss factors influencing the selection of a suitable material for a machine element. **(10 Marks)**
    c. Explain the codes and standards used in machine design. **(05 Marks)**

Dec. 2013/Jan. 2014 (10ME52)

24. a. Explain:
 i. Mechanical engineering design
 ii. Standards in design. **(04 Marks)**
 b. Explain:
 A point in a structural member is subjected to plane stress as shown in **Fig. U1.5.** Determine the principal stresses and their directions. **(08 Marks)**

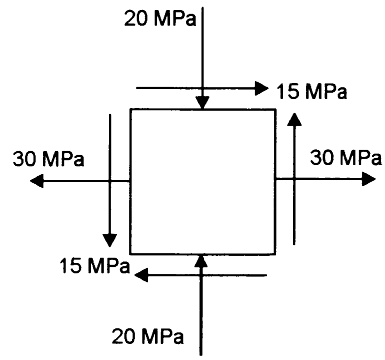


Fig. U1.5

June/July 2014 (10ME52)

25. a. Draw the stress–strain diagram for a ductile material and a brittle material and show the salient points on them. **(05 Marks)**
 b. For the stress element shown in **Fig. U1.6,** find the principal stresses and directions. **(06 Marks)**

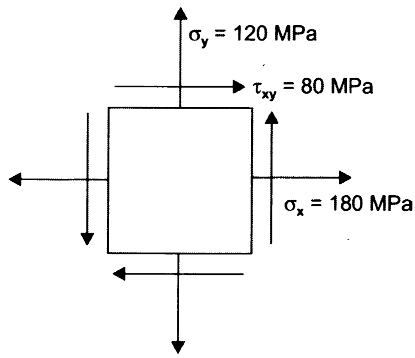


Fig. U1.6

Dec. 2014/ Jan. 2015 (06ME52)

26. a. Explain the material code for any two engineering materials. **(02 Marks)**
 b. Explain in brief, any six important factors governing the selection of materials for a machine member. **(06 Marks)**

Dec. 2014/ Jan 2015 (10ME52)

27. Briefly discuss three dimensional stress field and stress tensor. **(10 Marks)**

June/July 2015 (06ME52)

28. a. Draw stress–strain diagram for mild steel. Name the salient points. **(05 Marks)**
 b. Explain with a neat sketch, biaxial and triaxial stresses. **(05 Marks)**
 c. A point in a structural member subjected to plane stress is shown in **Fig. U1.7**. Determine the following.
 i. Normal and tangential stress intensities on plane MN inclined at 45°.
 ii. Principal stresses and their directions.
 iii. Maximum shear stress and the directions on the planes on which it occurs. **(10 Marks)**

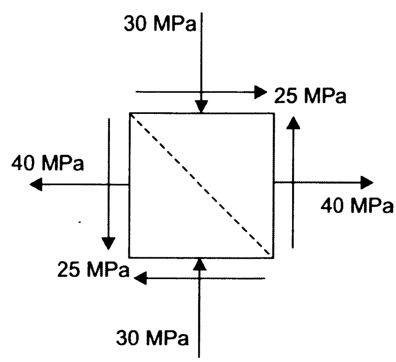


Fig. U1.7

June/July 2015 (10ME52)

29. a. What is mechanical engineering design? State the steps involved in mechanical engineering design. **(04 Marks)**
 b. Explain biaxial and triaxial stresses with neat sketches. **(04 Marks)**

Dec. 2015/ Jan. 2016 (10ME52)

30. a. Explain briefly the selection of factor of safety in engineering design. **(03 Marks)**

b. Explain briefly the selection of materials in the process of machine design. **(03 Marks)**

31. A point in a structural member is subjected to plane state of stress as shown in **Fig. U1.8**. Determine the following:

i. Normal and tangential stresses on a plane inclined at 45°.

ii. Principal stresses directions

iii. Maximum shear stress. **(07 Marks)**

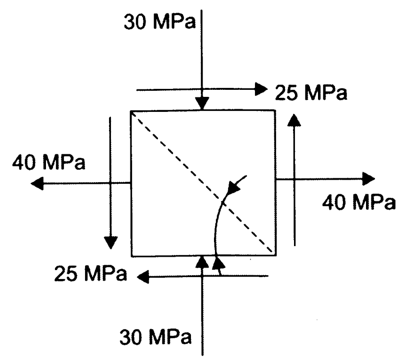


Fig. U1.8