Contents

1.	Some Mathematical Preliminaries and Convex Sets	1-38
1.	Introduction	1
1.:	2 Matrix	1
1.3	3 Type of Matrices	1
1.4	Operation on Matrices	3
1.	5 Properties of Matrix Addition	5
1.0	Properties of Multiplication of Matrix by a scalar	6
1.	Multiplication of Matrices	7
1.8	B Determinant of a Square Matrix	8
1.9	Properties of Determinants	8
1.10	Evolution of a Determinant by Sarrus diagram	9
1.1	Minors and Cofactors	9
1.12	2 Singular and Non-Singular Matrix	10
1.1;	3 Transpose of a Matrix	10
1.14	Properties of Transpose of a Matrix	11
1.1	Symmetric Matrix	12
1.10	Skew-symmetric Matrix	12
1.1	Rank of a Matrix	12
1.10	Echelon form of a Matrix	13
1.1	P Inverse of a Matrix	14
1.20	Convex Set	1/
1.2	Some Related Definitions	1/
1.2	Convex Function and Convex Polyhedron	19
1.2	Gonvex Function and Convex Polynedron	19
1.24		30
Z .	Inear Programming Problems: Formulation and Graphical solutions	39-94
2.		39
2.2	2 Basic Terminology of Linear Programming	39
Z.,	Basic Requirements of LPP	40
2.4	Basic Assumptions of LP model	40
2.3	Advantage of Linear Programming	40
2.0	Application areas of Linear Programming	41
2.	Application aleas of Linear Programming Problems	41
2.0	Matrix form of LPP	42 // 12
2.1	Mathematical Formulation of Linear Programming Problem	43
2.1	Extra Variable needed	43
2.1	2 Solution of Linear Programming Problem	53
2.1	Graphical solution methods of an LPP	54
3.	Simplex Method	95-160
3	Introduction	95
3.3	2 Terminology and Notations	95
0.1		

34	Fundamental Theorem of Linear Programming	96
2 5	Reduction of Feasible Solution to Basic Feasible Solution	98
3.0 3.6	Conditions for the Existence of Unbounded Solutions	110
3.0	Conditions for Improved Basic Feasible Solution to become Ontimal	112
3.8	Alternative optimal solutions	114
3.9	Inconsistency and Redundancy	114
3.10	Procedure to obtain Initial Basic Feasible Solution	114
3.11	Simplex Algorithm	116
3.12	Simplex Method : Case of minimization	130
3.13	Artificial Variable Technique	130
3.14	Two Phase Method	139
3.15	Some special Linear Programming Problems	144
3.16	Solution of Simultaneous Linear Equations by Simplex Method	153
3.17	Inverse of a Matrix by Simplex Method	155
4. De	egeneracy in Linear Programming	161-174
4.1	Introduction	161
4.2	Degeneracy in Linear Programming	161
4.3	The necessary and Sufficient Condition for the	
	Existence of Non-Degeneracy	161
4.4	Occurance of Degeneracy in Linear Programming	161
4.5	Resolution of Degeneracy	161
	avised Simpley Method	476 046
5. Re	evised omplex method	1/5-210
5. Re 5.1	Introduction	175-216
5. R 5.1 5.2	Introduction Standard Form of Revised Simplex Method	175-216 175 175
5. R 5.1 5.2 5.3	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I	175-216 175 175 175
5. R 5.1 5.2 5.3 5.4	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II	175-216 175 175 175 193
5. R 5.1 5.2 5.3 5.4 5.5	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method	175-216 175 175 175 193 213
5. R 5.1 5.2 5.3 5.4 5.5 6. D	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method uality in Linear Programming	175-216 175 175 175 193 213 271-252
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method uality in Linear Programming Introduction	175-216 175 175 175 193 213 271-252 217
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Itality in Linear Programming Introduction Relationship Between Prime and Dual	175-216 175 175 193 213 271-252 217 217
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems	175-216 175 175 193 213 271-252 217 217 218
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions	175-216 175 175 193 213 271-252 217 217 218 220
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality	175-216 175 175 193 213 271-252 217 217 218 220 231
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jul Simplex Method and Primal-Dual Algorithm	175-216 175 175 193 213 271-252 217 217 217 218 220 231 253-272
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jul Simplex Method and Primal-Dual Algorithm Introduction	175-216 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.1 7.2	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jul Simplex Method and Primal-Dual Algorithm Introduction Dual-Simplex Algorithm	175-216 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253 253
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.2 7.3	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Julity in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jul Simplex Method and Primal-Dual Algorithm Introduction Dual-Simplex Algorithm Primal-Dual Algorithm	175-216 175 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253 253 253 262
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.1 7.2 7.3 7.4	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Iterative in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Introduction Introduction Introduction Dual-Simplex Algorithm Primal-Dual Algorithm Steps of Primal-Dual Algorithm	175-216 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253 253 253 253 262 266
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.2 7.3 7.4 8. Se	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Uality in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Ual Simplex Method and Primal-Dual Algorithm Introduction Dual-Simplex Algorithm Primal-Dual Algorithm Steps of Primal-Dual Algorithm	175-216 175 175 175 193 213 271-252 217 217 217 217 218 220 231 253-272 253 253 253 262 266 273-304
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.2 7.3 7.4 8. Se 8.1	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Jality in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jal Simplex Method and Primal-Dual Algorithm Introduction Dual-Simplex Algorithm Primal-Dual Algorithm Steps of Primal-Dual Algorithm Ensitivity Analysis Introduction	175-216 175 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253 253 253 253 262 266 273-304 273
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.2 7.3 7.4 8. S 8.1 8.2	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Jality in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jal Simplex Method and Primal-Dual Algorithm Introduction Dual-Simplex Algorithm Primal-Dual Algorithm Steps of Primal-Dual Algorithm Steps of Primal-Dual Algorithm Introduction Introduction Change in the Objective Function Coefficients (price vectors), c _i	175-216 175 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253 253 253 253 262 266 273-304 273
5. R 5.1 5.2 5.3 5.4 5.5 6. D 6.1 6.2 6.3 6.4 6.5 7. D 7.1 7.2 7.3 7.4 8. Se 8.1 8.2 8.3	Introduction Standard Form of Revised Simplex Method Revised Simplex method for Standard form-I Revised Simplex method for Standard form-II Comparison of Simplex Method and Revised Simplex Method Jality in Linear Programming Introduction Relationship Between Prime and Dual Symmetric Primal-Dual Problems Dual of an LPP with Mixed Restrictions Some Results on Duality Jal Simplex Method and Primal-Dual Algorithm Introduction Dual-Simplex Algorithm Primal-Dual Algorithm Steps of Primal-Dual Algorithm Steps of Primal-Dual Algorithm Introduction Change in the Objective Function Coefficients (price vectors), c _j Variation in the Requirement Vector, b.	175-216 175 175 175 193 213 271-252 217 217 217 218 220 231 253-272 253 253 253 253 266 273-304 273 273 273 282

8.4 8.5 8.6	Variation in the Elements a _{ij} of the Coefficient Matrix A Addition of a New Variable Addition of a New Constraint	289 292 292
9. P	arametric Linear Programming	305-332
9.1 9.2 9.3 9.4	Introduction Parametric Programming Systematic Variation in the Objective function Coefficients, c _j Systematic Linear variation in b _i	305 305 305 323
10. In	teger Programming	333-364
10.1 10.2 10.3 10.4 10.5 10.6	Introduction Need of Integer Linear Programming Types of Integer Linear Programming Problems Methods to solve an Integer Linear Programming Problem Problem on Mixed Integer Linear Programming The Branch and Bound technique	333 333 333 334 348 354
11. TI	ne Transportation Problem	365-424
11.1 11.2 11.3 11.4 11.5	Introduction Mathematical Formulation of Transportation Problem Solution of the Transportation Problem Test for Optimality Degeneracy in Transportation Problems	365 365 369 385 398
11.6 11.7 12. A 12.1	Unbalanced Transportation Problem Some Miscellaneous Solved Problems ssignment Problems	405 408 425-466 425
11.6 11.7 12. A 12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 12.9	Unbalanced Transportation Problem Some Miscellaneous Solved Problems ssignment Problems Introduction Mathematical Representation of Assignment Problem Difference between Transportation and Assignment Problem Theorems on Assignment Problem Solution of Assignment Problem : Hungarian Method The Maximal Assignment Problem Unbalanced Assignment Problem Some Miscellaneous Solved Examples Travelling Salesman Problem	405 408 425-466 425 425 426 426 426 427 438 439 442 455
11.6 11.7 12. A 12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 12.9 13. G	Unbalanced Transportation Problem Some Miscellaneous Solved Problems ssignment Problems Introduction Mathematical Representation of Assignment Problem Difference between Transportation and Assignment Problem Theorems on Assignment Problem Solution of Assignment Problem : Hungarian Method The Maximal Assignment Problem Unbalanced Assignment Problem Some Miscellaneous Solved Examples Travelling Salesman Problem	405 408 425-466 425 425 426 426 427 438 439 442 455 467-506
11.6 11.7 12. A 12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 12.9 13. G 13.1 13.2 13.3 13.4 13.5	Unbalanced Transportation Problem Some Miscellaneous Solved Problems ssignment Problems Introduction Mathematical Representation of Assignment Problem Difference between Transportation and Assignment Problem Theorems on Assignment Problem Solution of Assignment Problem : Hungarian Method The Maximal Assignment Problem Unbalanced Assignment Problem Some Miscellaneous Solved Examples Travelling Salesman Problem oal Programming Goal Programming Model Formulation General form of Goal Programming Problem Method of Solution of a GP Problem	405 408 425-466 425 425 426 426 427 438 439 442 455 467-506 467 467 467 467 467 473 474
11.6 11.7 12. A 12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 12.9 13. G 13.1 13.2 13.3 13.4 13.5 14. G	Unbalanced Transportation Problem Some Miscellaneous Solved Problems ssignment Problems Introduction Mathematical Representation of Assignment Problem Difference between Transportation and Assignment Problem Theorems on Assignment Problem Solution of Assignment Problem : Hungarian Method The Maximal Assignment Problem Unbalanced Assignment Problem Some Miscellaneous Solved Examples Travelling Salesman Problem Oal Programming Goal Programming Model Formulation General form of Goal Programming Problem Method of Solution of a GP Problem	405 408 425-466 425 425 426 426 427 438 439 442 455 467-506 467 467 467 467 467 467 467 467 467

14.3	Properties of Optimal Mixed Strategies	516
14.4	Solution of 2 x 2 Games Without Saddle Points	516
14.5	Solution by linear programming	522
14.6	Minimax Theorem : Fundamental theorem of Game Theory	523
14.7	Dominance Property	529
14.8	Graphical Method for the Solution of 2 × n and m × 2 Games	536
14.9	Algebraic Method for the solution of a General Game	545
Bibliography		549
Index		551-552