



Contents

Preface *v*

1. Computer Arithmetic and Errors	1
1.1 Introduction	1
1.2 Floating Point Representation of Numbers	1
1.2.1 Normalization	2
1.3 Arithmetic Operations with Normalized Floating Point Numbers	2
1.3.1 Addition	2
1.3.2 Subtraction	4
1.3.3 Multiplication	4
1.3.4 Division	5
1.4 Error Analysis	9
1.5 Numbers and Their Accuracy	9
1.6 Significant Digits	9
1.7 Source of Errors	10
1.8 Rounding off	11
1.9 Chopping	12
1.10 Types of Errors	13
1.11 Machine Epsilon	14
1.12 General Formula for Terms	14
1.13 Errors in Arithmetic Operations	15
1.14 Numerical Instability	22
2. Roots of Equations	23
2.1 Introduction	23
2.2 Solution of an Equation	23
2.3 Mean Value Theorem	23
2.4 Taylor's Theorem	23
2.5 Methods of Finding Roots (Zeros) of an Equation	24
2.6 Order of Convergence of Iterative Methods	24

2.7 Bisection Method or Bolzano's Method	24
2.7.1 Convergence of Bisection Method	25
2.7.2 Algorithm for Bisection Method	26
2.8 Regula–Falsi Method or Method of False Position	29
2.8.1 Convergence of Regula–Falsi Method	29
2.8.2 Algorithm for Regula–Falsi Method	29
2.9 Secant Method	31
2.9.1 Rate of Convergence of Secant Method	32
2.9.2 Algorithm for Secant Method	34
2.10 Fixed Point Iteration Method or Method of Successive Approximation	38
2.10.1 Condition for Convergence of Fixed Point Iteration Method	39
2.10.2 Order of Convergence of Fixed Point Iteration Method	39
2.10.3 Algorithm for Fixed Point Iteration Method	40
2.11 Aitken's Δ^2 Method	43
2.12 Newton–Raphson Method	46
2.12.1 Geometrical Explanation of Newton–Raphson's Method	47
2.12.2 Criterion of the Convergence in Newton–Raphson's Method	47
2.12.3 Order of Convergence of Newton–Raphson's Method	48
2.12.4 Algorithm for Newton–Raphson Method	49
2.13 Modified Newton–Raphson Method for Multiple Roots	56
2.14 Polynomial Equations	58
2.14.1 Some Facts About Polynomial Equations	58
2.15 Birge–Vieta Method for Polynomials	58
2.15.1 Algorithm for Birge–Vieta Method	60
2.16 Lin–Bairstow's Method for Quadratic Factor	64
2.16.1 Algorithm for Lin–Bairstow's Method	66
3. Calculus of Finite Differences	72
3.1 Introduction	72
3.2 Differences	72
3.2.1 Forward Difference Operator	72
3.2.2 Backward Difference Operator	73
3.2.3 Shift Operator	73
3.2.4 Central Difference Operator δ	74
3.2.5 Averaging Operator μ	74
3.2.6 Relation between Operators	74
3.3 Factorial Notation	75
3.3.1 Differences of Factorial Notation	75
3.3.2 Generalized Factorial Notation	77
3.4 Leibnitz's Rule for Differences	77
3.5 Differences of Zero	77
3.5.1 Recurrence Relation for Differences of Zero	78
4. Interpolation	82
4.1 Introduction	82
4.2 Gregory–Newton Forward Interpolation Formula	82
4.3 Gregory–Newton Backward Interpolation Formula	83

4.4 Central Difference Interpolation Formulae	88
4.5 Gauss's Forward Interpolation Formula	88
4.6 Gauss's Backward Interpolation Formula	89
4.7 Stirling's Interpolation Formula	91
4.8 Bessel's Interpolation Formula	91
4.9 Laplace–Everett's Interpolation Formula	92
4.10 Lagrange's Interpolation Formula	97
4.10.1 Algorithm for Lagrange's Interpolation Formula	98
4.10.2 Different Form of Lagrange's Interpolation Formula	98
4.11 Divided Difference Interpolation	102
4.11.1 Newton's Divided Difference Interpolation Formula	102
4.11.2 Algorithm for Newton's Divided Difference Interpolation Formula	103
4.12 The Error in Polynomial Interpolation: Introduction	109
4.12.1 Error in Taylor Series Interpolation	109
4.12.2 Truncation Error or Remainder Term	110
4.12.3 Error Bound	110
4.12.4 Weierstrass Approximation Theorem	110
4.12.5 Roll's Theorem	110
4.12.6 Truncation Error in Lagrange Interpolation Polynomial	110
4.12.7 Truncation Error in Divided Difference Interpolation Formula	111
5. Piecewise and Spline Interpolation	120
5.1 Spline Interpolation	120
5.2 Piecewise Linear Interpolation	120
5.3 Cubic Spline Interpolation	122
6. Approximation of Functions	129
6.1 Introduction	129
6.2 Weierstrass Approximation Theorem	129
6.3 Weight Functions	130
6.4 Linearly Independent and Dependent Functions	131
6.5 Least Square Approximation	131
6.6 Algorithm for Fitting a Straight Line of the Form $y = a + bx$ for a Given Set of Data Points	132
6.7 Derivation of Normal Equations for Least Square Approximation for Discrete Data	137
6.7.1 Transformation of Data to Linear Form	137
6.8 Ill Conditioned System and Use of Orthogonal Functions	147
6.8.1 Orthogonal Functions	147
6.9 Gram–Schmidt Process of Orthogonalization	149
6.10 Chebyshev Polynomials	152
6.10.1 Orthogonal Properties	154
6.10.2 Properties of Chebyshev Polynomial	154
6.11 Uniform Approximation	159

x Computer Based Numerical and Statistical Techniques

6.12 Uniform (or Minimax) Polynomial Approximation	159
6.13 Chebyshev Equioscillation Theorem	160
6.14 Chebyshev Polynomial Approximation	162
6.15 Lanczos Economization of Power Series for a General Function	164
7. Numerical Differentiation	170
7.1 Introduction	170
7.2 Methods Based on Interpolation	170
7.3 Methods Based on Finite Differences	173
7.4 Richardson Extrapolation Method	184
8. Numerical Integration	191
8.1 Introduction	191
8.2 General Quadrature Formula	191
8.3 Trapezoidal Rule	192
8.3.1 Truncation Error in Trapezoidal Rule	192
8.4 Simpson's 1/3rd Rule	193
8.4.1 Truncation Error in Simpson's 1/3rd Rule	194
8.5 Simpson's 3/8th Rule	195
8.5.1 Truncation Error in Simpson's 3/8th Rule	196
8.6 Boole's Rule	205
8.6.1 Truncation Error in Boole's Rule	205
8.7 Weddle's Rule	207
8.8 Numerical Integration	210
8.9 Newton–Cote's Integration Method	211
8.9.1 Particular Cases	212
8.9.2 Closed Type Methods	213
8.10 Gaussian Integration Method	214
8.11 Gauss–Legendre Integration Method	215
8.12 Lobatto Integration Method	217
8.13 Radau Integration Method	219
9. Solution of Simultaneous Linear Algebraic Equations	237
9.1 Introduction	237
9.2 Direct Methods	237
9.3 Iterative Methods	237
9.4 Gauss Elimination Method	237
9.5 Gauss–Jordan Elimination Method	244
9.6 Triangularization method	247
9.7 Ill Conditioned System of Equations	251
9.7.1 Ill Conditioned Matrix	251
9.7.2 Improve Accuracy of Ill Conditioned System	251
9.8 Vector Norm	254
9.9 Matrix Norm	255

9.10 Condition Number 256	
9.10.1 Properties of Condition Number 256	
9.11 Successive Over Relaxation (SOR) Method 261	
9.11.1 Convergence of SOR Method 262	
9.12 Jacobi Iterative Method or Method of Simultaneous Displacements 267	
9.13 Gauss–Seidel Method 270	
10. Solution of Ordinary Differential Equations	278
10.1 Introduction 278	
10.2 Picard’s Method 278	
10.3 Taylor’s Series Method 279	
10.4 Euler’s Method 280	
10.4.1 Error Estimates for the Euler Method 280	
10.5 Modified Euler’s Method 281	
10.6 Runge–Kutta Methods 291	
10.6.1 Runge–Kutta First Order Method 292	
10.6.2 Runge–Kutta Second Order Method 292	
10.6.3 Runge–Kutta Third Order Method 293	
10.6.4 Runge–Kutta Fourth Order Method 293	
10.6.5 Runge–Kutta Method for Simultaneous First Order Differential Equations 301	
10.6.6 Runge–Kutta Method for Second Order Differential Equation 302	
10.7 Predictor–Corrector Methods 306	
10.8 Milne’s Method 306	
10.9 Adam–Bashforth (or Adam’s) Predictor–Corrector Methods 313	
11. Algebraic Eigen Values and Eigen Vectors of Matrices	320
11.1 Introduction 320	
11.2 Eigen Values and Eigen Vectors 320	
11.3 Gerschgorin Circle Theorem 323	
11.4 Jacobi’s Method 324	
11.5 Given’s Method 325	
11.6 House–Holder’s Method 335	
11.7 Dominant Latent Root (Rayleigh’s Power Method) 336	
11.8 QR Method 336	
11.9 Reduction of a Matrix to Upper Hessenberg Matrix 337	
12. Statistical Techniques	350
12.1 Introduction 350	
12.2 Population or Universe 350	
12.3 Sample 350	
12.4 Statistical Hypothesis 350	
12.5 Parameters of Statistics 351	
12.6 Hypothesis Testing 351	
12.7 Error in Testing of Hypothesis 352	

12.8 Two Tailed and One Tailed Tests of Hypothesis	353
12.9 Standard Error	353
12.10 Test of Significance for Large Sampling	353
12.11 Z-test	354
12.11.1 Testing the Significance of Population Proportion	354
12.11.2 Testing the Significance of the Difference in Proportion	358
12.11.3 Testing the Significance of Population Mean	361
12.11.4 Testing the Significance of the Difference between Two Sample Mean or Population Mean	363
12.11.5 Testing the Significance of Two Sample Standard Deviations	367
Appendix	368
Index.....	371