hardened concrete of the member itself, which has been precast a few weeks in advance, the technique is called post-tensioning. In this case, the jacks can be released only after the prestress of the high tensile steel is transferred as a compressive force in concrete by means of end anchorages.

In all prestressed members, whether pretensioned or post-tensioned, the two main elements are the high tensile steel which is initially in tension and concrete which carries an equal and opposite compressive force. Thus the two main elements, viz. the prestressing high tensile steel and the concrete together constitute a self equilibrating system. As the two elements induce or develop prestress by straining each other, a prestressed concrete member may be visualised as a case of self straining. While the resultant forces in the two elements are equal in magnitude the manner in which they carry these forces are quite different. The high tensile wires, bars, multiwire strands and cables have negligible stiffness againt shear, bending and torsional deformations. Hence high tensile prestressing steel essentially carries only axial tension. The total tensile force in all tendons may be assumed to act along the axis of an imaginary cable which coincides with the line, called the cable line, connecting the centroids of the tensile forces in individual tendons at all cross-sections over the entire length of the member. The other element, i.e. concrete, has appreciable stiffness against the deformations caused by the four types of internal forces, viz. the axial and shear forces and the bending and twisting moments. Consequently, it is capable of resisting all four types of internal forces. The stress distribution in concrete must be such that it provides stress resultants equal and opposite to the four internal forces at every cross-section of the member.

As the prestressing steel is in a state of simple tension, free from shear forces and bending and twisting moments, it is easy to draw the free body diagram by considering the equilibrium of the free body of prestressing steel. Thus all forces acting on the cable can be readily computed if the axial tension is known. The forces imposed on concrete by the prestressing steel are equal in magnitude and opposite in sign to those acting on prestressing steel. The net forces on concrete can be calculated by combining these forces, which are caused only due to the effect of prestress, with the external forces. Once these net forces are known, the structure may be analysed for stresses like any ordinary non-prestressed structure. The effect of prestress on the stresses and deformation of the structure is fully taken care of by considering the forces imposed by the prestressing tendons on the surrounding concrete. This approach, known as the free body approach, is very useful in the analysis of internal forces, particularly in statically indeterminate structures.

1.4 SIGN CONVENTION

The sign convention adopted throughout this book conforms to those most widely utilised in structural analysis and have, therefore, received almost universal acceptance. The tensile forces and the corresponding tensile stresses and strains are taken to be positive while the compressive forces, stresses and strains are treated as negative quantities. As a prestressed concrete member represents a self-straining system, the prestressing force exerts a tensile force on prestressing steel and an equal compressive

In the case of transversely loaded prestressed concrete beam having total prestressing force P with eccentricity e_p, the resultant bending moment M_r may be expressed as

$$M_r = M_p + M = -\ Pe_p + M \tag{1.7.3}$$

in which M is the external bending moment. As the only axial force at any cross-section of the beam is the prestressing force, the resultant compression $C = P$. Hence from Eq. (1.7.2), the eccentricity of the centre of compression in a prestressed concrete beam may be expressed as

$$e_c = -\frac{M_r}{C} = -\frac{M_r}{P} = e_p - \frac{M}{P} \tag{1.7.4}$$

In an externally unloaded beam having $M = 0$,

$$e_c = e_p \tag{1.7.5}$$

Equation (1.7.5) shows that in an unloaded prestressed concrete beam, the line of thrust concides with the cable line. As the external moment M is applied, the line of thrust shifts from the cable line. The vertical intercept between the line of thrust and the cable line at any cross section is equal to (M/P). When M becomes numerically equal to M_p but is opposite in sign, i.e. for $M = -\ M_p$, the eccentricity of line of thrust $e_c = 0$. Hecne for $M = + M_p$, the line of thrust coincides with the centroidal axis of the member.

The position of the centre of compression or the line of thrust is important because the stresses at any cross-section depend on it. In a prestressed concrete beam with total compression $C = P$, the stress f at any point at a distance y from the centroidal axis may be determined from

$$= -\frac{P}{A} + \frac{(P\,e_c)\,y}{I} \tag{1.7.6}$$

in which A and I are the cross-sectional area and its moment of inertia respectively. The first term on the right of Eq. (1.7.6) represents the direct stress due to P. The second term represents the bending stress caused by the resultant bending moment $M_r = M_p + M = -\ P\,e_c$ in accordance with Eq. (1.7.1). The value of e_c at any section of the beam can be determined readily from Eq. (1.7.4).

EXAMPLE 1.7.1

Determine the position of the cable line at midspan on the basis of gross area in the beam of Example 1.5.1 assuming that the prestress in all wires is 1100 MPa. Hence determine the position of centre of compression if the external sagging bending moment acting at the section is 0, 20, 32.40 and 40 kN.m respectively. Comment on the position of centre of compression for different values of bending moment.

Solution

$$\text{Prestressing force in each wire} = \frac{\pi}{4} \times 5^2 \times 1100 \times 10^{-3}$$

$$= 21.61 \text{ kN}$$

It is interesting to note the contrast in the structural responses of prestressed and reinforced concrete beams. In a prestressed concrete beam, C remains constant while jd_p increases proportionately with M. On the contrary, in a reinforced concrete section shown in Fig. 1.10.2 (a), the internal lever arm (jd_p) remains practically constant because the position of the neutral axis and the centre of compression located at the centroid of the stress diagram shown in Fig. 1.10.2 (b) remain practically unchanged during the entire working range. Consequently, the total compression in concrete C and total tension in reinforcing steel T increase proportionately with the applied bending moment M. The stress in the steel in a reinforced concrete beam varies from zero to the full working stress as the load increases from zero to the full working or service load. On the other hand, the steel in a prestressed concrete beam carries practically constant stress when the load increases from zero to the full working load. It is, therefore, evident that prestressing steel is less prone to fatigue failure as compared to reinforcing steel. An interesting aspect of the active and passive roles of prestressing steel and reinforcing bars is worth mentioning. In a reinforced concrete beam, an increase in reinforcing steel can do no harm unless it leads to congestion creating difficulty in placement and compaction of concrete. On the other hand, even a small increase in the prestressing steel and the consequent increase in active prestressing force may create unacceptable stresses and deflections. Hence prestressing steel has to be designed more carefully than reinforcing steel.

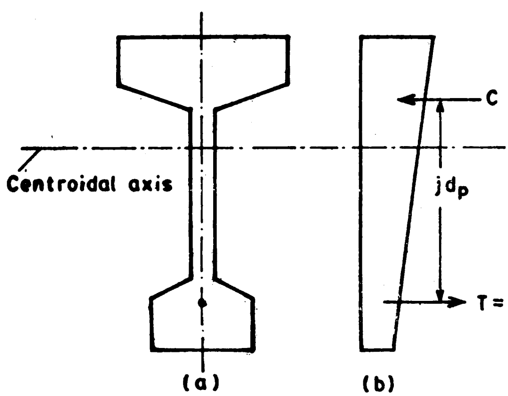

Fig. 1.10.1

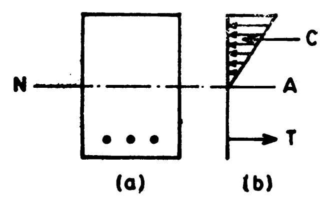

Fig. 1.10.2

1.11 KERN POINTS

The kern points in the cross-section of a compression member are the points representing extreme positions of the centre of compression without producing tensile stress at the extreme top or bottom fibres. When the centre of compression moves to the upper extreme limit, called the upper kern point, the compressive stress at the bottom fibre f_B reduces to zero. Similarly, when the centre of compression moves to the lower extreme limit, called the lower kern point, the compressive stress at the extreme top fibre f_T reduces self weight, effective at the time of prestressing, may be counteracted by merely increasing the eccentricity of prestressing force without any increase in the area of the prestressing steel. Thus a prestressed concrete girder may carry the dead load without any extra cost. As the dead loads dominate over the live loads with increasing span, the prestressed concrete construction shows spectacular economy in the field of long span structures.

(xii) *Economy*

Due to more efficient use of concrete and reductions in shear force and deflection, the prestressed concrete members are much slender as compared to reinforced concrete members. The net saving of concrete may be of the order of 50 to 60 per cent. The saving in the volume of steel due to the use of high tensile steel may be as high as 80 per cent. Thus there is a significant saving in both concrete and steel. Although some economy due to prestressing is possible in almost every field of concrete construction, a spectacular economy is achieved in the domain of long span construction. In bridge construction with spans ranging from 30 m to 100 m, prestressed concrete may compare favourably with reinforced concrete and structural steel. In the field of large containment structures, prestressed concrete may provide the most economical design. In the case of supersized tanks, prestressed concrete is not only the most economical but perhaps the only feasible option.

When prestressed concrete was first introduced successfully, it was thought that it will make reinforced concrete obsolete. It did not do so. Nor is it likely to happen in the future. Actually reinforced concrete and prestressed concrete play complementary roles. There are many areas in which reinforced concrete is more appropriate as a material of construction. The main disadvantages of prestressed concrete are as follows:

(a) *Complexity of design*

The complexity of prestressed concrete design arises mainly from the multiplicity of the loading stages for which the design must be checked. Besides, the prestressing force being an active force unlike the passive force offered by the reinforcing bars, an excess of prestressing force may be as unsafe as the lack of it. Hence the prestressing force has to be critically designed.

(b) *Skilled supervision and control*

Prestressed concrete calls for a much higher skilled supervision and control as compared to reinforced concrete. The need for high grade concrete, the higher dimensional accuracy and the smaller tolerances permitted in prestressed concrete construction demand very close supervision. The operations associated with the prestressing require experience and expertise. The need for a high quality control also demands a high degree of supervision.

(c) *Costly equipment*

Considerable investment on tools and plant is necessary in prestressed concrete construction. The equipment depends upon the system of prestressing used. Although the cost of the equipment varies substantially for different