where γ = propagation constant = $\alpha + j\beta = \sqrt{Z \cdot Y}$
α = attenuation constant (N/m)
β = phase constant (rad/unit length)

Equations (2.22) and (2.23) yields standard linear differential equation as

$$V = V_+ e^{-\gamma z} + V_- e^{rz} \quad ...(2.24)$$

$$I = I_+ e^{-\gamma z} + I_- e^{\gamma z} \quad ...(2.25)$$

where V_+ and I_+ indicates complex amplitude in positive z direction, V_- and I_- signify complex amplitudes in negative z-direction.

2.2.2 Forward Impedance and Backward Admittance Travelling Waves

When a source (V, I) is applied at input of a transmission line, it propagates along the transmission lines. The voltage and current at any point of transmission line is the summation of transmitted and reflected wave due to mismatch between transmission line and load. Transmitted voltage waves or current wave and reflected voltage or current wave can by represented as,

$$V = V\text{ transmitted} + V\text{ reflected}$$

$$= A \cdot e^{-\gamma z} + Be^{+\gamma z}$$

or

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} \quad ...(2.26)$$

$\therefore$ forward and backward current

$$I = Ae^{-\gamma z} + Be^{\gamma z}$$

$$I = I_+ e^{-\gamma z} + I_- e^{\gamma z} \quad ...(2.27)$$

First terms of Eqs (2.26) and (2.27) are called forward travelling waves and second terms are called backward travelling wave. These two component are shown in Fig. 2.5.

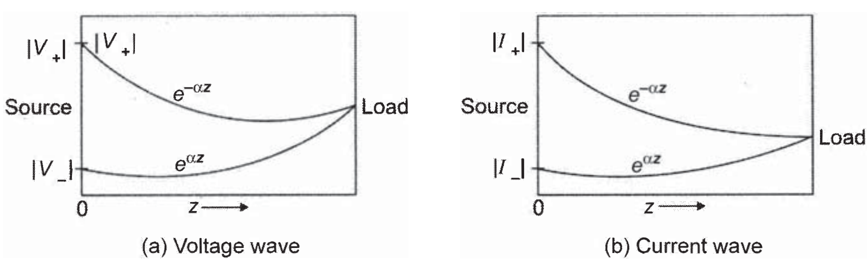

Fig. 2.5: Magnitude of voltage and current standing waves

Equations (2.26) and (2.27) are solutions of transmission line equations [Eqs (2.14) and (2.15)] and a schematic for transmission terminated with load is shown in Fig. 2.6.

The impedance of a transmission line is a complex ratio of the voltage phasor and current phasor at any point. It is defined as

$$Z \equiv \frac{V(z)}{I(z)} \quad ...(2.28)$$

Similarly for current

$$\text{ISWR} = \frac{I_{max}}{I_{min}} = \left[\frac{V_+e^{-\alpha z} + I_-e^{\alpha z}}{V_+e^{-\alpha z} - I_-e^{\alpha z}}\right] = \frac{I_+e^{-\alpha z}}{I_+e^{-\alpha z}}\left[\frac{1+|\Gamma|}{1-|\Gamma|}\right] \qquad \text{...(2.70a)}$$

Case 1 : When $V_- = 0$, $V_+ \neq 0$, reflected wave is zero, then

$$V_0 = \sqrt{(V_+e^{-\alpha})^2 \cos^2 \beta z + (V_+e^{-\alpha z})^2 \sin^2 \alpha z} = V_+e^{-\alpha z} \qquad \text{...(2.71)}$$

Case 2 : When $V_+ = 0 V_- \neq 0$

$$V_0 = V_-e^{\alpha z} \qquad \text{...(2.72)}$$

Case 3 : When both the positive and negative waves have equal amplitude or $|V_+e^{-\alpha z}| = |V_-e_+{}^{\alpha z}|$ magnitude of the reflection coefficient is unity and zero phase, it is called *pure standing wave*. It is given by

$$V_s = 2V_+e^{-\alpha z} \cos \beta z \qquad \text{...(2.73)}$$

Similarly for current $\quad I_s = -2j\, V_+e^{-\alpha z} \sin \beta z \qquad$ ...(2.74)

Equations (2.73) and (2.74) show the current and voltage standing waves are 90° out of phase along the line and also ISWR and VSWR are identical in magnitude as in Fig. 2.10.

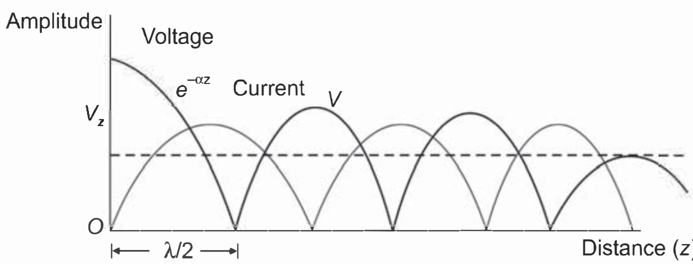

Fig. 2.10: Pure standing waves of voltage and current

2.3 SMITH CHART

Smith chart is the graphical representation of a transmission line. All the transmission parameters can be calculated with the help of this chart. It is also known as *impedance calculator*, as polar impedance diagram, consists of two sets of circles. The Smith chart consists of a plot of the normalized impedance or admittance with the angle and magnitude of a complex reflection coefficient in a unit circle. It is applicable for lossless as well as lossy line. To see how Smith chart works, consider the equation at reflection coefficient at the load as

$$\Gamma = \frac{Z_l - Z_0}{Z_l + Z_0}$$

or

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0} = \Gamma_r + \Gamma_i \qquad \text{...(2.75)}$$

Since $|\Gamma_l| \leq$ so the value of Γ_l must lie on or within unit circle with a radius of 1. Let

$$Z = R + jX$$

$$\frac{Z}{Z_0} = \frac{R + jX}{Z_0} = r + jx = z \text{ (normalized impedance)} \qquad \text{...(2.76)}$$