Differentiating Eq. 10.14, the rate of change of velocity is given by:

$$\frac{\partial v}{\partial z} = \frac{k}{p_w} \frac{\partial^2 \bar{u}}{\partial z^2} \quad (10.15)$$

If v is the velocity of water at entry into the soil element, the velocity at exit is equal to $v + \frac{\partial v}{\partial z} \Delta z$

The quantity of water entering the soil element per unit time is $v \Delta x \Delta y$ and the quantity of water which flows out of the element is $(v + \frac{\partial v}{\partial z} \Delta z) \Delta x \Delta y$. The net quantity of water or the excess pore water Δq squeezed out of the soil element per unit time is, therefore, given by:

$$\Delta q = \frac{\partial v}{\partial z} \Delta x \Delta y \Delta z \quad (10.16)$$

As excess water flows out, the volume of the soil element decreases, the decrease in volume being equal to the volume of water flowing out.

From Eq. 10.7 the change in volume ΔV of the soil element is given by:

$$\Delta V = - m_v V_i \Delta\sigma' \quad (10.17)$$

where: m_v = coefficient of volume change, assumed constant
V_i = initial volume of soil element
$= \Delta x \Delta y \Delta z$

The change in volume per unit time is given by:

$$\frac{\partial (\Delta V)}{\partial t} = - m_v \Delta x \Delta y \Delta z \frac{\partial (\Delta\sigma')}{\partial t} \quad (10.18)$$

Equating expressions 10.16 and 10.18 and cancelling $\Delta x \Delta y \Delta z$:

$$\frac{\partial v}{\partial z} = - m_v \frac{\partial (\Delta\sigma')}{\partial t} \quad (10.19)$$

Also, the differentiation of Eq. 10.11, where $\Delta\sigma$ is constant, leads to

$$\frac{\partial (\Delta\sigma')}{\partial t} = - \frac{\partial \bar{u}}{\partial t} \quad (10.20)$$

Eq. 10.19 may, therefore, be written as:

$$\frac{\partial v}{\partial z} = m_v \frac{\partial \bar{u}}{\partial t} \quad (10.21)$$

Combining Eq. 10.21 and 10.15:

$$m_v \frac{\partial \bar{u}}{\partial t} = \frac{k}{p_w} \frac{\partial^2 \bar{u}}{\partial z^2}$$

$$\rho = \int_0^H m_v (\Delta\sigma - \bar{u})\, dz$$

or

$$\rho = m_v \left[\Delta\sigma H - \int_0^H \bar{u}\, dz\right]$$

Substituting the value of $\bar{u}$ from Eq. 10.44 and integrating:

$$\rho = m_v \Delta\sigma H \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} e^{-\frac{(2N+1)^2 \pi^2}{H^2} c_v t}\right] \tag{10.46}$$

At $t = \infty$, when the process of consolidation is complete, the *ultimate* or the *final* settlement ρ_f is given by:

$$\rho_f = m_v \Delta\sigma H \tag{10.47}$$

The final settlement ρ_f can also be obtained directly from Eq. 10.10. After a very great time when $\bar{u}$ reduces to zero, $\Delta\sigma'$ equals the consolidating pressure $\Delta\sigma$ which is the same throughout the depth of the layer for the case illustrated in Fig. 10.4. Therefore, the final settlement due to consolidation can be directly written in the form of Eq. 10.47.

The ratio of ρ to ρ_f, expressed as a percentage, is termed the *degree of consolidation* U:

$$U\,(\%) = \frac{\rho}{\rho_f} \times 100 \tag{10.48}$$

or

$$U\,(\%) = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} e^{-\frac{(2N+1)^2 \pi^2}{H^2} c_v t}\right] \times 100 \tag{10.49}$$

Introducing a dimensionless parameter called the *time factor* T_v defined by the following equation

$$T_v = \frac{c_v t}{d^2} \tag{10.50}$$

where: d = *drainage path* ($= \frac{H}{2}$, for the case under consideration)

Eq. 10.49 may be written as:

$$U\,(\%) = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} e^{-\frac{(2N+1)^2 \pi^2}{4} T_v}\right] \times 100 \tag{10.51}$$

or

$$U = f(T_v) \tag{10.52}$$

The above simplified relation (Eq. 10.52) obtained from Terzaghi's theory of consolidation shows that the degree of consolidation is a func-

The factor of safety of an infinite slope of a cohesionless soil is thus independent of the depth of the assumed failure plane.

(*b*) *Cohesive Soil.* The failure envelope DE_2 for a soil possessing both cohesion and friction as given by Eq. 12·7 ($\tau^f = c' + \sigma' \tan \phi'$) is shown in Fig. 14.3. If the slope angle β ($=\beta_1$) is less than ϕ', no critical state of stress is reached at any depth and the slope remains stable. When β ($=\beta_2$) is greater than ϕ', the line OE_2 (Fig. 14.3) intersects the failure envelope in E_2 and a state of incipient failure is reached because the shear stress τ corresponding to the depth represented by the point E_2 equals the shear strength τ_f. For greater depths, the shear stress is greater than the shear strength and the slope cannot remain stable.

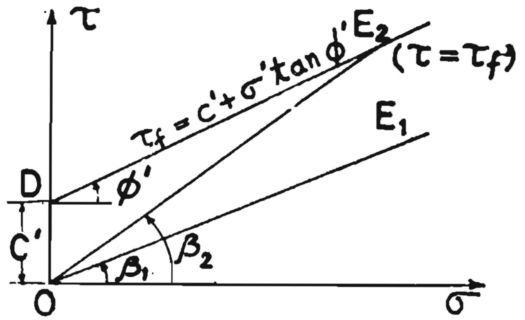

FIG. 14.3

Failure Condition for an Infinite Slope of Cohesive Soil

Thus an infinite slope of a semi-infinite mass of soil possessing both cohesion and friction cannot remain stable if β is greater than ϕ'. If β is greater than ϕ' the slope can be stable only upto a limited depth, known as the *critical depth*, and which is calculated as given below.

The factor of safety against failure is given by:

$$F = \frac{\tau_f}{\tau} = \frac{c' + \gamma z \cos^2\beta . \tan\phi'}{\gamma z \cos\beta \sin\beta} \tag{14.4}$$

When compared to Eq. 14.3, it can be seen from Eq. 14.4 that the factor of safety for cohesive soils depends not only on the angles ϕ' and β but also on γ, c' and the depth z. The critical depth $z = H_c$ is obtained when τ equals τ_f (point E_2 in Fig. 14.3) or the factor of safety is unity.

From Eq. 14.4, the critical depth (or height) may be written as:

$$H_c = \frac{c'}{\gamma} \frac{1}{(\tan\beta - \tan\phi') \cos^2\beta} \tag{14.5}$$

Driving moment $M_D = W\bar{x}$ (14.11)

Resisting moment $M_R = c_u\hat{L}r$ (14.12)

where: $\hat{L}$ = length of arc AD $\left(=\dfrac{2\pi r\omega^\circ}{360}\right)$ (14.13)

The factor of safety F against sliding is then given by:

$$F = \frac{M_R}{M_D} = \frac{c_u\hat{L}r}{W\bar{x}} \tag{14.14}$$

Alternatively, let c_m be the mobilised shear resistance of soil ($\phi_u = 0$) necessary for equilibrium of the assumed sliding wedge of soil, then:

$$W.\bar{x} = c_m\hat{L}r$$

$$c_m = \frac{W\bar{x}}{\hat{L}r} \tag{14.15}$$

and factor of safety $$F = \frac{c_u}{c_m} \tag{14.16}$$

The critical slip surface (the critical circle) corresponds to the minimum factor of safety. When F corresponding to the critical circle is less than unity, the slope is considered unstable. When the minimum F equals unity, failure is just impending. For a stable slope, the minimum F should generally be equal to or greater than 1.5.

To find the distance $\bar{x}$ of the line of action of the weight W from the centre of rotation, the sector $ADBA$ may be divided into parallel vertical strips (*See* method of slices below) and the moment of each strip taken about O. Alternatively, the centre of gravity G is determined experimentally by cutting a piece of thin cardboard to the shape of the sector and suspending it in turn from two points The verticals when drawn through the points of suspension intersect at the centre of gravity.

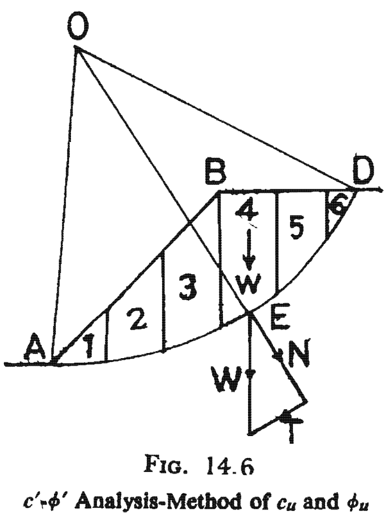

FIG. 14.6

c'-ϕ' Analysis-Method of c_u and ϕ_u

(*b*) $c' - \phi'$ *Analysis.* The shear strength of a soil with an angle of shearing resistance ϕ' varies with the effective normal pressure on the slip surface. For such a soil, the stability analysis is carried out by the *method of slices* in which the soil profile inside the assumed slip circle is divided into a convenient number of vertical strips or slices, as shown in Fig. 14.6. The