CHAPTER 1

Introduction to Signals and Systems

1.1 Signals

(GPR, Nov' 16, 1 Mark & GTU, Jan' 16, 7 Marks)

Any physical phenomenon that conveys or carries some information can be called a signal.

Examples: Music, speech, motion pictures, still photos, heart beat, etc.

Signal: *A physical quantity that varies with one or more independent variables is called a signal.*

The independent variables can be time, spatial coordinates, intensity of colours, pressure, temperature, etc. The most popular independent variable in signals is time and it is represented by the letter "t".

Amplitude: *The value of a signal at any specified value of the independent variable is called its amplitude.*

Waveform: *The sketch or plot of the amplitude of a signal as a function of the independent variable is called its waveform.*

Mathematically, any signal can be represented as a function of one or more independent variables.

Table 1.1: Examples of Signals

Basis for Classification	Type	Definition	Example
Number of sources	One-channel signals	Signals that are generated by a single source.	i) Record of room temperature. ii) Audio output of a monospeaker.
	Multi-channel signals	Signals that are generated by multiple sources.	i) Record of ECG at eight different places in a human body. ii) Audio output of two stereo speakers.
Number of dimensions	One-dimensional signals	Signal which is a function of a single independent variable.	i) Music, speech and heart beat which are function of a single independent variable, time. ii) $x_1(t) = 0.7t$.
	Multi-dimensional signals	Signal which is a function of two or more independent variables.	i) Photograph is two-dimensional (2D) signal. ii) Motion picture of a black and white TV is a three-dimensional (3D) signal.

Table 1.1: Continued...

Basis for Classification	Type	Definition	Example
Whether the dependent variable is continuous or discrete	Analog or continuous signal	Signal which is defined continuously for any value of the independent variable is called **analog signal**. When the independent variable of an analog signal is time, it is called **continuous time signal.**	Most of the signals encountered in science and engineering are analog.
	Discrete signal	Signal which is defined for discrete intervals of the independent variable is called **discrete signal.** When the independent variable of a discrete signal is time, it is called **discrete time signal.**	Sampled version of analog signal.

1.1.1 Continuous Time Signal

Continuous time signal: *A signal which is defined continuously for any value of the independent variable time "t" is called continuous time signal and it is denoted as "x(t)".*

Example: Sinusoidal signal, $x(t) = A \sin \Omega_0 t$

where, $\Omega_0 = 2\pi F = 2\pi/T$

The continuous time signal is defined for every instant of the independent variable time and so the magnitude (or the value) of continuous time signal is continuous in the specified range of time. Here both the magnitude of the signal and the independent variable are continuous.

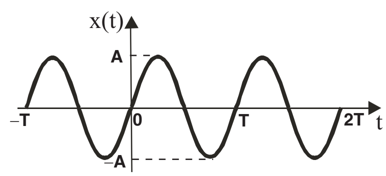

Fig 1.1: *Sinusoidal signal.*

1.1.2 Discrete Time Signal

Discrete time signal: *A signal which is defined only for discrete instants of the independent variable time is called discrete time signal and it is denoted as "x(n)".*

Example: $x(n) = \{-1.6, -0.8, \underset{\uparrow}{0}, 0.8, 1.6\}$

A discrete time signal is sampled version of continuous time signal. In discrete time signal the independent variable time "t" is uniformly divided into discrete intervals of time and each interval of time is denoted by an integer "n", where "n" stands for discrete interval of time and "n" can take any integer value in the range $= -\infty$ to $+\infty$. Therefore, the magnitude of the discrete time signal is defined only for integer values of independent variable "n".

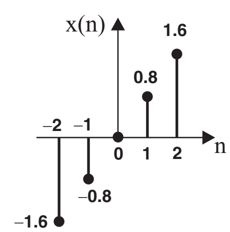

Fig 1.2: *A discrete time signal x(n).*

1.1.3 Digital Signal

Digital signal: *The quantized and coded version of a discrete time signal is called digital signal.*

In a digital signal the value of the signal for every discrete time "n" is represented in binary codes. The process of conversion of a discrete time signal to digital signal involves quantization and coding.

Normally, for binary representation, a standard size of binary is chosen. In m-bit binary representation we can have 2^m binary codes. The possible range of values of the discrete time signals are usually divided into 2^m steps called ***quantization levels***, and a binary code is attached to each quantization level. The values of the discrete time signals are approximated by rounding or truncation in order to match the nearest quantization level.

1.2 Systems

(GTU, Jan' 16, 7 Marks) (GPR, Nov' 16, 1 Mark)

System: *Any process that exhibits a cause and effect relation can be called a system.*

A system will have an input signal and an output signal. The output signal will be a processed version of the input signal. A system is interconnection of either hardware devices or software/algorithm. A system is denoted by the letter '$\mathcal{H}$'. The diagrammatic representation of a system is shown in Fig 1.3.

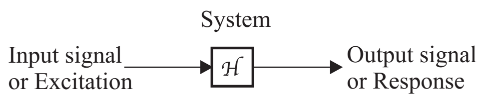

Fig 1.3: *Representation of a system.*

The operation performed by a system on the input signal to produce the output signal can be expressed as,

$$\text{Output} = \mathcal{H}\{\text{Input}\}$$

where $\mathcal{H}$ denotes the system operation (also called ***system operator***).

The systems can be classified in many ways.

Depending on the type of energy used to operate the systems, the systems can be classified into electrical systems, mechanical systems, thermal systems, hydraulic systems, etc.

Depending on the type of input and output signals, the systems can be classified into Continuous time systems and Discrete time systems.

1.2.1 Continuous Time System

Continuous time system: *A system which process and produce continuous time signal is called continuous time system.*

The input and output signals of a continuous time system are continuous time signals. The diagrammatic representation of a continuous time system is shown in Fig 1.4.

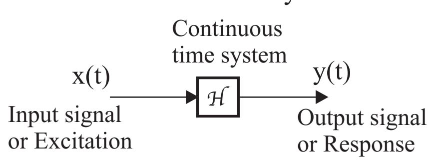

Fig 1.4: *Representation of a continuous time system.*

where, $\mathcal{H}$ = System operator

x(t) = Continuous time input signal

y(t) = Continuous time output signal

The operation performed by a continuous time system on input x(t) to produce output or response y(t) can be expressed as,

$$\text{Response,} \quad y(t) = \mathcal{H}\{x(t)\} \qquad .....(1.1)$$

Linear Time Invariant Continuous Time (LTI-CT) System: *A continuous time system which satisfies the properties of linearity and time invariance is called a Linear Time Invariant Continuous Time (LTI-CT) system.*

Most of the practical systems that we encounter in science and engineering are LTI systems.

Mathematical Representation of LTI-CT System

The input-output relation of an LTI continuous time system is represented by constant coefficient differential equation shown below:

$$a_0 \frac{d^N}{dt^N} y(t) + a_1 \frac{d^{N-1}}{dt^{N-1}} y(t) + a_2 \frac{d^{N-2}}{dt^{N-2}} y(t) + ... + a_{N-1} \frac{d}{dt} y(t) + a_N y(t) = b_0 \frac{d^M}{dt^M} x(t)$$

$$+ b_1 \frac{d^{M-1}}{dt^{M-1}} x(t) + b_2 \frac{d^{M-2}}{dt^{M-2}} x(t) + ... + b_{M-1} \frac{d}{dt} x(t) + b_M x(t) \qquad .....(1.2)$$

where, N = Order of the system, $M \leq N$, $a_0 = 1$.

$a_1, a_2, ..., a_N, b_1, b_2, ..., b_M$ are constant coefficients

The solution of the above differential equation is the response y(t) of the continuous time system, for the input x(t).

1.2.2 Discrete Time System

Discrete time system: *A system which processes an input discrete time signal and producean output discrete time signal is called discrete time system.*

The input and output signals of a discrete time system are discrete time signals. The diagrammatic representation of a discrete time system is shown in Fig 1.5.

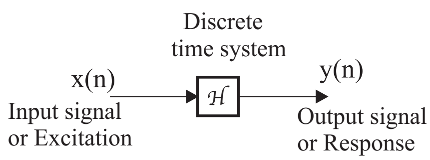

Fig 1.5: *Representation of a discrete time system.*

where, $\mathcal{H}$ = System operator

x(n) = Discrete time input signal

y(n) = Discrete time output signal

The operation performed by a discrete time system on input x(n) to produce output or response y(n) can be expressed as,

Response, $y(n) = \mathcal{H}\{x(n)\}$ .....(1.3)

Linear Time Invariant Discrete Time (LTI-DT) System: *A discrete time system which satisfies the properties of linearity and time invariance is called a Linear Time Invariant Discrete Time (LTI-DT) system.*

Mathematical representation of LTI-DT system

The input-output relation of an LTI discrete time system is represented by the constant coefficient difference equation shown below:

$$y(n) = -\sum_{m=1}^{N} a_m\, y(n-m) + \sum_{m=0}^{M} b_m\, x(n-m) \qquad .....(1.4)$$

where, N = Order of the system, $M \leq N$.

a_m, b_m are constant coefficients.

The solution of the above difference equation is the response y(n) of the discrete time system, for the input x(n).

1.3 Frequency Domain Analysis of Continuous Time Signals and Systems

Physically, we can realise any signal or system in time domain. In the time domain, continuous time systems are governed by differential equations. The analysis of continuous time signals and systems in the time domain involves solution of differential equations. The solution of differential equations is difficult to achieve by making an assumption for a solution and then solving the constants using initial conditions.

In order to simplify the task of analysis, the signals can be transformed to some other domain, where the analysis is easier. One such transform which exists for continuous time signals is Laplace transform. The ***Laplace transform***, will transform a function of time "t" into a function of complex frequency "s", where $s = \sigma + j\Omega$. Therefore, Laplace transform of a continuous time signal will transform the time domain signal into an s-domain signal.

On taking Laplace transform of the differential equation governing the system, it becomes an algebraic equation in "s" and the solution of the algebraic equation will give the response of the system as a function of "s" and it is called an s-domain response. The inverse Laplace transform of the s-domain response will give the time domain response of the continuous time system. Also, stability analysis of continuous time systems is much easier in s-domain.

Another important characteristic of any signal is frequency and for most applications the frequency content of a signal is an important criteria. The frequency contents of a signal can be studied by taking Fourier transform of a signal. The Fourier transform of a signal is a particular class of Laplace transform in which $\sigma = 0$ and $s = j\Omega$, where "Ω" is real frequency.

The ***Fourier transform*** will transform a function of time "t" into a function of real frequency "Ω". Therefore, Fourier transform of a continuous time signal will transform the time domain signal into a frequency domain signal. From the Fourier transform of a continuous time signal, the frequency spectrum of the signal can be obtained. This is used to study the frequency content of a signal. The frequency range of some signals are listed in Table 1.2 and 1.3.

Table 1.2: Frequency Range of some Biological and Seismic Signals

Type of Signal	Frequency Range (Hz)
Electroretinogram	0 to 20
Electronystagmogram	0 to 20
Pneumogram	0 to 40
Electrocardiogram (ECG)	0 to 100
Electroencephalogram (EEG)	0 to 100
Electromyogram	10 to 200
Sphygmomanogram	0 to 200
Speech	100 to 4000
Wind noise	100 to 1000
Seismic exploration signals	1 to 100
Earthquake and nuclear explosion signals	0.01 to 1
Seismic noise	0.1 to 1

Table 1.3: Frequency Range of some Electromagnetic Signals

Type of Signal	Wavelength (m)	Frequency Range (Hz)
Radio broadcast	10^4 to 10^2	3×10^4 to 3×10^6
Shortwave radio signals	10^2 to 10^{-2}	3×10^6 to 3×10^{10}
Radar/Space communications	1 to 10^{-2}	3×10^8 to 3×10^{10}
Common-carrier microwave	1 to 10^{-2}	3×10^8 to 3×10^{10}
Infrared	10^{-3} to 10^{-6}	3×10^{11} to 3×10^{14}
Visible light	3.9×10^{-7} to 8.1×10^{-7}	3.7×10^{14} to 7.7×10^{14}
Ultraviolet	10^{-7} to 10^{-8}	3×10^{15} to 3×10^{16}
Gamma rays and x-rays	10^{-9} to 10^{-10}	3×10^{17} to 3×10^{18}

1.4 Frequency Domain Analysis of Discrete Time Signals and Systems

Mostly, discrete time systems are designed for analysis of discrete time signals. Physically, discrete time systems can also be realised in the time domain. In the time domain, discrete time systems are governed by difference equations.

The analysis of discrete time signals and systems in time domain involves solution of difference equations. The solution of difference equations is difficult to achieve by making an assumption for a solution and then solving the constants using initial conditions.

In order to simplify the task of analysis, the discrete time signals can be transformed to some other domain, where the analysis may be easier. One such transform which exists for discrete time signals is Ƶ-transform. The Ƶ-transform will transform a function of discrete time "n" into a function of complex variable "z" where, $z = re^{j\omega}$. Therefore, ***Ƶ-transform*** of a discrete time signal will transform the time domain signal into a z-domain signal.

On taking Ƶ-transform of the difference equation governing the discrete time system, it becomes an algebraic equation in "z" and the solution of the algebraic equation will give the response of the system as a function of "z" and it is called a z-domain response.The inverse Ƶ-transform of the z-domain response will give the time domain response of the discrete time system. Also, stability analysis of discrete systems are much easier in z-domain.

The frequency contents of a discrete time signal can be studied by taking Fourier transform of the discrete time signal. The Fourier transform of a discrete time signal is a particular class of Ƶ-transform in which $z = e^{j\omega}$, where "ω" is the frequency of the discrete time signal.

The Fourier transform will transform a function of discrete time "n" into a function of frequency "ω". Therefore, Fourier transform of a discrete time signal will transform the discrete time signal into a frequency domain signal. From the Fourier transform of the discrete time signal, the frequency spectrum of the discrete time signal can be obtained. This is used to study the frequency content of the discrete time signal.

1.5 Importance of Signals and Systems

Every part of the universe is a system which generates or processes some type of signal [of course the universe itself is a system and said to be controlled by signals (or commands) issued by God].

Signals and systems play a vital role in almost every field of science and engineering. Some applications of signals and systems in various fields of science and engineering are listed ahead:

1. Biomedical

- ECG is used to predict heart diseases.
- EEG is used to study normal and abnormal behaviour of the brain.
- EMG is used to study the condition of muscles.
- X-ray images are used to predict bone fractures and tuberculosis.
- Ultrasonic scan images of kidney and gall bladder are used to predict presence of stones.
- . Ultrasonic scan images of foetus are used to predict presence of abnormalities in a baby.
- MRI scan is used to study minute inner details of any part of the human body.

2. Speech Processing

- Speech compression and decompression is used to reduce memory requirement of storage systems.
- Speech compression and decompression is used for effective use of transmission channels.
- Speech recognisation is used for voice-operated systems and voice-based security systems.
- Speech recognisation is used for conversion of voice to text.
- Speech synthesis is used for various voice-based warnings or annoucements.

3. Audio and Video Equipments

- Analysis of audio signals is useful to design systems for special effects in audio systems like stereo, woofer, karoke, equalizer, attenuator, etc.
- Music synthesis and composing is done using music keyboards.
- Audio and video compression is used for storage in DVDs.

4. Communication

- Spectrum analysis of modulated signals helps to identify the information bearing frequency component that can be used for transmission.
- Analysis of signals received from radars is used to detect flying objects and their velocity.
- Generation and detection of DTMF signals in telephones.
- Echo and noise cancellation in transmission channels.

5. Power Electronics

- Spectrum analysis of the output of converters and inverters reveals the harmonics present in the output, which in turn helps to design suitable filters to eliminate the harmonics.
- Analysis of switching currents and voltages in power devices helps to reduce losses.

6. Image Processing

- Image compression and decompression is used to reduce memory requirement of storage systems.
- Image compression and decompression is used for effective use of transmission channels.
- Image recognisation is used for security systems.
- Filtering operations on images are done to extract the features or hidden information.

7. Geology

- Seismic signals are used to determine the magnitude of earthquakes and volcanic eruptions.
- Seismic signals are also used to predict nuclear explosions.
- Seismic noise is used to predict the movement of the earth's layers (tectonic plates).

8. Astronomy

- Analysis of light received from a star is used to determine the condition of the star.
- Analysis of images of various celestial bodies gives vital information about them.

1.6 Use of MATLAB in Signals and Systems

The ***MATLAB*** (**MAT**rix **LAB**oratory) is a software developed by The MathWork Inc, USA, which can run on any windows platform in a PC (Personal Computer). It has a number of tools for the study of various engineering subjects and includes a tool for signal processing as well. A wide variety of studies can be done on signals and systems using this tool. Some analysis that is relevant to this particular textbook is given below:

- Sketch or plot of signals as a function of an independent variable.
- Spectrum analysis of signals.
- Solution of LTI systems.
- Performing convolution and deconvolution operations on signals.
- Performing various transforms on signals like Laplace transform, Fourier transform, $\mathcal{Z}$-transform, Fast Fourier Transform (FFT), etc.
- Determination of state model from transfer function and viceversa.
- Stability analysis of signals and systems in various domains.