$$K = \frac{\sigma_c^2 \sin\psi_n \cos\psi_n \left(\frac{1}{E_p} + \frac{1}{E_g}\right)}{1.4}$$

σ_c = Surface endurance strength (N/mm^2)

E_p and E_g being modulus of elasticity for pinion and gear respectively.

ψ_n = Normal pressure angle

For steel gears with 20° normal pressure angle (ψ_n)

'K' modifies to $K = 0.16\left(\frac{BHN}{100}\right)^2$

Wear strength (S_w) indicates the maximum tangential force that the tooth can transmit without pitting failure.

For proper design, wear strength (S_w) should be more than the effective force between the meshing teeth.

2.4 DESIGN OF HELICAL GEAR

For design of helical gears, effective load on gear tooth needs to be determined, both for beam strength as well as wear strength as design criteria.

• In the preliminary stages of design, effective tooth load 'P_{eff}' between 2 teeth in mesh is given by:

$$P_{eff} = \frac{C_S P_t}{C_V}$$

where C_S = Service factor = Operating torque

and C_V = Velocity factor $= \frac{5.6}{5.6 + \sqrt{V}}$ (for helical gears)

where V = Peripheral velocity of gear $= \left(\frac{\pi d N}{60}\right)$

(d = Pitch circle diameter, N = rpm)

and P_t = Tangential component of the resultant force between two meshing teeth of helical gears.

and is obtained as $\rightarrow P_t = \frac{2M_t}{d}$ (d = Pitch circle diameter)

where M_t = Transmitted torque

$$= \left(\frac{60 \times 10^6 \times P_{\text{kW}}}{2\pi N}\right) \qquad [P_{\text{kW}} = \text{Power in kilowatt}]$$

• In final stages of gear design, after rough determination of gear dimensions, quality of gears and error specification, 'dynamic load' on gear is determined by following equation:

$$\boxed{P_d = \frac{21V\left(Ceb\cos^2\alpha + P_t\right)\cos\alpha}{21V + \sqrt{\left(Ceb\cos^2\alpha + P_t\right)}}}$$

α = Helix angle (degrees)

P_d = Dynamic load (N)

V = Pitch line velocity (m/s)

So overlap should be around 15% of circular pitch for proper design.

So overlap = $b \tan\alpha = 1.15\ P_c \Rightarrow\ b = \dfrac{1.15\pi m}{\tan\alpha}$

where b = Minimum face width and

m = Module

3. A pair of helical gears are used to transmit 18 kW at 8000 rpm of the pinion. The teeth are 20° stub in diametral plane and the helix angle is 45°. The gear and the pinion have a pitch diameter of 320 mm and 80 mm respectively. Both gear and pinion are made up of cast steel with allowable static strength of 100 MPa, suggest a suitable module and face width for the gear pain and check the strength of the design in wear. Take modulus of elasticity for cast steel as 2×10^5 MPa and σ_{es} = 618 MPa. [UPTU 2006]

Solution: Given:

Power (P) = 18 kW

Speed of pinion (N_p) = 8,000 rpm

Pressure angle (ψ) = 20° stub tooth

Helix angle (α) = 45°, d_g = Pitch circle diameter of gear = 320 mm = 0.32 m

d_p = Pitch circle diameter of pinion = 80 mm = 0.08 m.

Since both pinion and gear are made up of cast steel

($\Rightarrow$ allowable static strength = 100 MPa

$E = 2 \times 10^5$ MPa, σ_{es} = Endurance strength = 618 MPa)

So pinion will be considered as weaker and design will be based upon it.

Torque transmitted by the pinion $(T) = \dfrac{P \times 60}{2\pi N_p}$

$$= \frac{18 \times 10^3 \times 60}{2\pi \times 8{,}000} = 21.49 \text{ N-m}$$

Tangential load on pinion

$$F_t = \frac{T}{d_p/2} = \frac{21.49}{0.08/2} = 537.25 \text{ N}$$

So number of teeth on pinion

$$T_p = \frac{d_p}{m} = \frac{80}{m}$$

Equivalent (formative) no. of teeth on pinion

$$(T_p)_E = \frac{T_p}{\cos^3\alpha} = \frac{80/m}{\cos^3(45)} = \frac{226.4}{m}$$

Tooth form factor for pinion for 20° stub tooth system: $y_p = 0.175 - \dfrac{9.95}{T_p}$

$$= 0.175 - \frac{9.95}{226.4/m} = (0.175 - 0.0042m)$$

Peripheral velocity $= \dfrac{\pi d_p N_p}{60} = \dfrac{\pi \times 0.08 \times 8{,}000}{60}$

$= 33.51$ m/sec

a. Pitch: Pitch (P_x) of worm is defined as the distance measured from a point on one thread to the corresponding point on the adjacent thread along the axis of worm.

b. Lead: Lead (l) of worm is defined as the distance that a point on thread will move, when worm is rotated through one revolution.

For single start threads, lead is equal to the axial pitch, whereas for double start threads, lead is twice the pitch.

So $$l = P_x z_1$$

Depending upon requirements of velocity ratio, single- or multi-threaded worm is used.

Single start	:	Velocity ratio 20 and above
double start	:	12–36
triple start	:	8–12
Quadrupole start	:	6–12
Sextuple start	:	4–10

(**Note:** Single-threaded worm refers to a thread length comprising of a single thread in one revolution whereas a multithread say triple start/threaded worm refers to a thread length comprising of three threads in one revolution.)

c. Lead angle: Lead angle (γ) is the angle between tangent to the thread at the pitch diameter and a plane normal to worm axis such that:

From Fig. 3.3.1,

$$\tan\gamma = l/\pi d_1$$

$$\Rightarrow \quad \tan\gamma = \frac{P_x z_1}{\pi(qm)} = \frac{\pi m z_1}{\pi(qm)} \quad (\text{as } P_x = \pi m)$$

$$\Rightarrow \quad \tan\gamma = z_1/q$$

d. Helix angle (Fig. 3.3): Helix angle (ψ) is the angle between a tangent to the thread at the pitch diameter and the axis of worm.

Helix angle should be limited to 6° per thread.

e. Pressure angle (Fig. 3.2): Tooth pressure angle (α) is equal to half of the thread angle and should not be less than 20° for single/double start worms and 25° for triple/multistart worms.

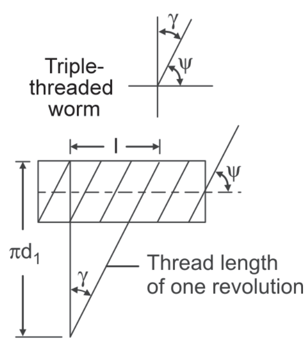

Fig. 3.3 Triple-threaded worm

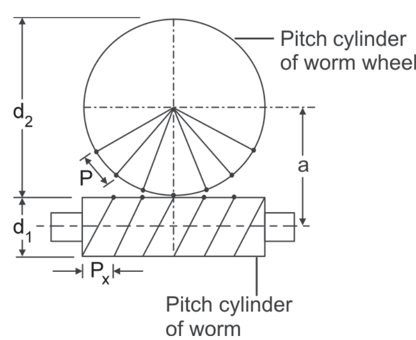

Fig. 3.4 Pitch cylinder (worm)

f. Center distance (a): Center distance between worm and worm wheel is given by:

$$a = \frac{1}{2} m (q + z_2)$$

q: Diametral quotient $= d_1/m$

z_2: Number of teeth on worm wheel

Axial pitch of threads on worm (P_a) = $\pi m = \pi \times 12 = 37.68$ mm
and axial lead of thread on worm

$$l = P_a \times n = 37.62 \times 2 = 75.36 \text{ mm}$$

Normal lead of thread on worm

$$l_N = l \cos \lambda = 80.95 \cos (22.1)$$
$$l_N = 75 \text{ mm}$$

also center distance $(x) = \dfrac{l_N}{2\pi}\left(\dfrac{1}{\sin\gamma} + \dfrac{V_r}{\cos\alpha}\right)$

$$x = \frac{75}{2\pi}\left(\frac{1}{\sin(22.1)} + \frac{15}{\cos(22.1)}\right) = 225 \text{ mm}$$

$$x = 225 \text{ mm} \Rightarrow \text{(result is correct)}$$

Let $\quad d_w$ = Pitch circle diameter of worm

also $\quad \tan \lambda = l/\pi d_w$

$$d_w = \frac{l}{\pi \tan\gamma} = \frac{80.95}{3.14 \times \tan(22.1)} = 63.48$$
$$d_w = 64 \text{ mm.}$$

Since velocity ratio is 15 and worm has double threads ($n = T_w = 2$), therefore, number of teeth on worm gear (T_g) = 15 × 2 = 30

We find that face length of worm or length of threaded portion (L_w) = P_c (4.5 + 0.2 × T_w) = 37.68 (4.5 + 0.2 × 2) = 171 mm ($P_c = P_a$)

This length should be increased by 25 to 30 mm for the feed marks produced by vibrating grinding wheel as it leaves thread root,

So taking L_w = 195 mm

also depth of tooth (h) = 0.623 P_c = 0.623 × 37.68 = 23.47 mm.

addendum (a) = 0.286P_c = 10.72 mm

Outside diameter of worm (d_{ow}) = $d_w + 2a$ = 63.48 + 2 × 10.72

So $\quad d_{ow}$ = 84.91 mm

Design of Worm Gear

Pitch circle diameter of worm gear

$$d_g = mT_g = 12 \times 30 = 360 \text{ mm.}$$

Outside diameter of worm gear

$$d_{og} = d_g + 0.8903\, P_c = 360 + 0.8903 \times 37.68$$
$$d_{og} = 393.6 \text{ mm.}$$

Throat diameter

$$d_T = d_g + 0.572\, P_c = 360 + 0.572 \times 37.68$$
$$d_T = 381.6 \text{ mm.}$$

and face width $\quad b = 2.15\, P_c + 5$ mm

or $\quad b = 2.15 \times 37.68 + 5 = 86$ mm

Checking of Designed Worm Gearing as per Tangential Load, Dynamic Load

a. Checking for tangential load, let N_g = Speed of worm gear in rpm.