(a) 272800 (b) 2728
(c) 272700 (d) none of these

26. The l.c.m. of 172 and 20 is
(a) 865 (b) 860
(c) 680 (d) none of these

27. The l.c.m. of 16, 10, 15 is
(a) 2450 (b) 2400
(c) 2500 (d) none of these

28. Luca's number is defined by
(a) $L_1 = 1$ (b) $L_2 = 3$
(c) $L_n = L_{n-1} + L_n - 2, \forall\ n > 2$
(d) all are true

29. Fibonacci's series is defined by
(a) $a_1 = 1$ (b) $a_2 = 1$
(c) $a_n = a_{n-1} + a_{n-2} \forall\ n > 2$
(d) all are true

30. If a_n is the nth term of the fibonacci's sequence and $\alpha = \dfrac{1+\sqrt{5}}{2}$ then
(a) $a_n > \alpha^{n-1} \forall\ n > 1$
(b) $a_n < \alpha^{n-1} \forall\ n > 1$
(c) $a_n > \alpha \forall\ n > 1$
(d) none of these

31. If n is an odd integer greater than 1 then $n^3 - n$ is a multiple of
(a) 24 (b) 26
(c) 25 (d) none of these

32. If a and b are positive integers such that $(a, b) = [a, b]$
(a) $a \neq b$ (b) $a = b$
(c) $a > b$ (d) $a < b$

33. If $(a, b) = 1$ then $(a - b, a + b)$
(a) 2 (b) 3
(c) 1 (d) 0

34. If $m > 0, n > 0$ and m is an odd integer, then
(a) $(2^m - 1)(2^n + 1) = 1$
(b) $(2^m - 1)(2^n + 1) = 0$
(c) $(2^m - 1)(2^{n+1}) = 1$
(d) none of these

35. If $(a, b) = 1$ then $(a + b, a^2 - ab + b^2) =$
(a) 1 (b) 2
(c) 3 (d) 0

ANSWERS

Fill in the blanks

(1) any integer c	(2) $\nmid b$	(3) 3	(4) not a	(5) an integer
(6) g.c.d.	(7) uniquely	(8) relatively prime	(9) 21	(10) 2

True/False

(1) T	(2) T	(3) F	(4) T	(5) T
(6) T	(7) T	(8) F	(9) T	(10) T

Multiple choice questions

(1) *a*	(2) *b*	(3) *c*	(4) *b*	(5) *a*	(6) *b*
(7) *a*	(8) *c*	(9) *d*	(10) *a*	(11) *a*	(12) *b*
(13) *a*	(14) *c*	(15) *a*	(16) *a*	(17) *a*	(18) *a*
(19) *b*	(20) *a*	(21) *a*	(22) *a*	(23) *a*	(24) *b*
(25) *a*	(26) *b*	(27) *b*	(28) *d*	(29) *a*	(30) *a*
(31) *a*	(32) *b*	(33) *c*	(34) *a*	(35) *a*	

CHAPTER REVIEW : A COMPETITIVE APPROACH

Selected terms and Results

TERMS

- **Inverse Sine of $(x + iy)$:** If $\sin(u + iv) = x + iy$ then $u + iv$ is called an inverse sine of $x + iy$.
- **Inverse cosine of $(x + iy)$:** If $\cos(u + iv) = x + iy$ then $u + iv$ is called an inverse cosine of $x + iy$.
- **Inverse hyperbolic function :** Let z and w be any two complex numbers. If $\sinh w = z$ then w is called the inverse hyperbolic sine of z.

RESULTS

- For general value of inverse sine, the first letter S being written capital.
- For principal value of inverse sine, the first letter 's' being written small and the real part lies between $-\frac{p}{2}$ and $\frac{p}{2}$.
- $\text{Sin}^{-1}(x + iy) = n\pi + (-1)^n \sin^{-1}(x + iy) : n \in Z$.
- $\text{Cos}^{-1}(x + iy) = 2n\pi \pm \cos^{-1}(x + iy) : n \in Z$.
- $\sinh^{-1} z = \log[z + \sqrt{z^2 + 1}]$
- $\cosh^{-1} z = \log[z + \sqrt{z^2 - 1}]$
- $\tanh^{-1} z = \frac{1}{2}\log\left[\frac{1+z}{1-z}\right]$.
- $\coth^{-1} z = \frac{1}{2}\log\left(\frac{z+1}{z-1}\right)$.
- $\sinh^{-1} x = -i\sin^{-1}(ix)$
- $\cosh^{-1} x = -i\cos^{-1} x$
- $\tanh^{-1} x = -i\tan^{-1}(ix)$

Review Questions and Project Work

1. Prove that $\sinh^{-1}(\cot x) = \log(\cot x + \text{cosec } x)$
2. Prove that $\sin^{-1}(\cot\theta + i\sin\theta) =$

   $\cos^{-1}\sqrt{\sin ?} + i\log(\sqrt{\sin ?} + 1) - \sqrt{\sin ?}$
3. If $\cos^{-1}(u + iv) = \alpha + i\beta$, prove that $\cos^2\alpha$ and $\cosh^2\beta$ are the roots of the equation $x^2 - (1 + u^2 + v^2)x + u^2 = 0$
4. If $\cosh^{-1}(x + iy) + \cosh^{-1}(x - iy) = \cosh^{-1} a$, show that

   $2(a - 1)x^2 + 2(a + 1)y^2 = a^2 - 1$
5. Prove that $\sinh^{-1}(\cot x) = \log(\cot x + \text{cosec } x)$

Objective Type Questions

Fill in the blanks :

1. $\sinh^{-1} z = \log$ [....................]
2. $\cosh^{-1} z = \log$ [....................]
3. $\sinh^{-1} x =$ .... $\sin^{-1}(ix)$
4. $\cosh^{-1} x =$ .... $\cos^{-1} x$
5. $\tanh^{-1} x =$ .... $\tan^{-1} x$

True/False: *Write 'T' for true and 'F' for false statement.*

1. $\tanh^{-1} x = i\tan^{-1}(ix)$ (T/F)
2. $\tanh^{-1} z = \frac{1}{2}\log\left[\frac{1+z}{1-z}\right]$ (T/F)
3. $\coth^{-1} z = \frac{1}{2}\log\left[\frac{z+1}{z-1}\right]$ (T/F)
4. $\sinh^{-1} z = \log[z + \sqrt{z^2 + 1}]$ (T/F)

Multiple Choice Questions : *Choose the most appropriate one :*

Now, we want to remove second term, then we shall equal to zero the coefficient of y^{n-1}, we get

$$na_0h + a_1 = 0 \quad \text{or} \quad h = -\frac{a_1}{na_0}$$

Hence we decreased all the roots of the given equation by a constant $-\frac{a_1}{na_0}$, the second term of the given equation can be removed.

Similarly if we want to remove third term, we put

$$\frac{n(n-1)}{2!}a_0h^2 + (n-1)a_1h + a_2 = 0$$

Solve this equation we get two values of h and similarly we can remove any term of the given equation.

(i) *To remove the second term of the equation*

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0$$

and form the equation with integral coefficients having leading coefficient unity.

Since the equation is

$$f(x) \equiv a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0 \quad \text{...(1)}$$

Let $\alpha_1, \alpha_2, \alpha_3$ be its roots

put $x = y + h$ in (1), we get

$$a_0(y+h)^3 + 3a_1(y+h)^2 + 3a_2(y+h) + a_3 = 0$$

or $\quad a_0(y^3 + h^3 + 3y^2h + 3yh^2) + 3a_1(y^2 + h^2 + 2yh) + 3a_2(y + h) + a_3 = 0$

or $\quad a_0y^3 + (3ha_0 + 3a_1)y^2 + (3h^2a_0 + 6a_1h + 3a_2)y + (a_0h^3 + 3a_1h^2 + 3a_2h + a_3) = 0 \quad$ ...(2)

Now we want to remove second term, then put

$$3ha_0 + 3a_1 = 0 \text{ or } h = -\frac{a_1}{a_0}$$

Substitute the value of h in (2), we get

$$a_0y^3 + \left(\frac{3a_1^2}{a_0} - \frac{6a_1^2}{a_0} + 3a_2\right)y + \left(-\frac{a_1^3}{a_0^2} + \frac{3a_1^2}{a_0^2} - \frac{3a_1a_2}{a_0} + a_3\right) = 0$$

or $$a_0y^3 + \frac{3(a_0a_2 - a_1^2)}{a_0}y + \frac{(a_0^2a_3 - 3a_0a_1a_2 + 2a_1^2)}{a_0^2} = 0$$

or $$a_0y^3 + \frac{3H}{a_0}y + \frac{G}{a_0^2} = 0 \quad \text{...(3)}$$

where $\quad \text{H} = a_0a_2 - a_1^2, \ \text{G} = a_0^2a_3 - 3a_0a_1a_2 + 2a_1^2$

Thus the equation (3) is a transformed equation. Further, make all the coefficients of (3) integers, so that (3) can be written as

Now substitute this value of x in (1), we get

$$\left(\frac{3r}{y+q}\right)^3 + q\left(\frac{3r}{y+q}\right) + r = 0$$

or $$(3r)^3 + 3qr(y+q)^2 + r(y+q)^3 = 0$$

or $$(y+q)^3 + 3q(y+q)^2 + 27r^2 = 0$$

$$y^3 + q^3 + 3y^2q + 3yq^2 + 3qy^2 + 3q^3 + 6q^2y + 27r^2 = 0$$

or $$y^3 + 6qy^2 + 9q^2y + (4q^3 + 27r^2) = 0$$

This is the required equation.

EXERCISE 12.1

1. Change the sign of the roots of the equation $x^5 - 4x^3 + 3x^2 + 8x - 9 = 0$
2. Transform the equation $x^3 - 4x^2 + \frac{1}{4}x - \frac{1}{9} = 0$ into another equation with integral coefficients and having leading coefficient unity.
3. Transform the equation $3x^4 - 5x^3 + x^2 - x + 1 = 0$ into another equation with integral coefficients having leading coefficient unity.
4. Find the equation whose roots are twice the reciprocals of the roots of
$$x^4 + 3x^3 - 6x^2 + 2x - 4 = 0$$
5. Remove the fractional coefficients from the equation
$$x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$$
6. Remove the fractional coefficients from the equation
$$x^4 - \frac{5}{6}x^3 - \frac{13}{12}x^2 + \frac{1}{300} = 0$$
7. Solve the following reciprocal equations :
   (i) $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$
   (ii) $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$
8. Reduce the equation $4x^4 - 85x^3 + 357x^2 - 340x + 64 = 0$ into a reciprocal equation.
9. Find the equation whose roots are the squares of the roots of the equation
$$x^4 + x^3 + 2x^2 + x + 1 = 0$$
10. Find the equation whose roots are the cubes of the roots of the following equations:
   (i) $x^3 + ax^2 + bx + ab = 0$ (ii) $x^3 + 3x^2 + 2 = 0$
11. Remove the second term form the following equations :
   (i) $x^3 - 6x^2 + 10x - 3 = 0$ (ii) $x^4 + 8x^3 + x - 5 = 0$
   (iii) $x^5 + 5x^4 + 3x^3 + x^2 + x - 1 = 0$ (iv) $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$
   (v) $x^6 - 12x^5 + 3x^2 - 17x + 300 = 0$