1.6.2 Number of Subsets of a Set

If A is a set contains n distinct element such that $0 < r \le n$. If we consider those subsets of A that have r elements each, then we know that the number of ways in which r elements can be choose out of n elements is ${}^nC_r$. Therefore, the number of subsets of A having r elements each is ${}^nC_r$.

Hence, the total number of subsets of A is equal to

$$ {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = (1+1)^n = 2^n $$

For example:

(1) If a set A has one element, then it has $2^1 = 2$ subsets.

(2) If a set A has two elements, then it has $2^2 = 4$ subsets.

REMARKS

☞ The number of proper subsets of a set with n elements is $2^n - 2$.

☞ The collection of all possible subsets of a given set A is called power set. It is denoted by $P(A)$. For example : If A = {1,2,3} then the power set $P(A)$ = { ϕ , {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}.

☞ $P(\phi) = \{\phi\}$

☞ The power set of any given set is always non-empty.

1.7 UNIVERSAL SET

In any discussion , we are given particular set and we consider different subsets of the given set. This given set is called Universal Set. It is denoted by U.

For Example:

(1) The universal set is of real numbers **R**, while considering the set of natural numbers, whole numbers, integers and rational numbers.

(2) The set of alphabets is the universal set from which the letters of any word may be chosen to form a set.

(3) In geometry, we discuss set of lines, triangles and circles, then the universal set is the plane, in which the lines, triangles and circles lie.

REMARKS

☞ Universal set is a super set of each of the given sets.

☞ The universal set is not unique.

1.7.1 Complement of a Set

Let U be the universal set and the set $A \subseteq U$. Complement of set A with respect to the universal set U is the set of all those elements of U which are not the elements of A and is denoted by A' or A^c,

i.e., $$A' = \{x : x \in U \text{ and } x \notin A\}$$

For example:

(i) If U= {1, 2, 3, 4, 5, 6, 7, 8, 9, 11} and A= {1, 2, 3}
then A'= {4, 5, 6, 7, 8, 9, 11}.

Therefore,

$A \sim B = \{x : x \in A \text{ and } x \notin B\} = A \cap B'$

And $B \sim A = \{x : x \notin A \text{ and } x \in B\} = B \cap A'$

For example:

Let $A = \{1, 2, 3, 4, 5\}$

And $B = \{-1, 0, 1, 2\}$

Then, $A \sim B = \{3, 4, 5\}$

And $B \sim A = \{-1, 0\}$

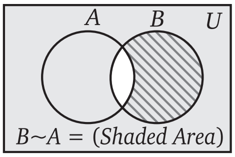

Fig. (8)

REMARKS

- ☞ $x \in (A - B) \Leftrightarrow x \in A$ and $x \notin B$.
- ☞ $x \notin (A - B) \Leftrightarrow x \notin A$ and $x \in B$
- ☞ $A - B \neq B \sim A$, *i.e.*, , difference of two sets is not commutative.
- ☞ $A \subset B$ then $A \sim B = \phi$
- ☞ The sets $A \sim B$, $A \cap B$ and $B \sim A$ are mutually disjoint.
- ☞ Difference of a set with the universal set is known as complementation.
- ☞ $A \sim B$ is a subset of A and $B \sim A$ is a subset of B.

1.9.4 Symmetric Difference of Two Sets

If A and B are two sets, then the symmetric difference of two sets A and B is denoted by $A \Delta B$ is given by $A \Delta B = (A \sim B) \cup (B \sim A)$

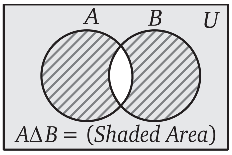

Fig. (9)

Symbolically:

$A \Delta B = \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

For example:

(i) If $A = \{1,2,3,4,5,6,7,8\}$ and $B = \{1,3,5,6,7,8,9\}$

Then $A \sim B = \{2, 4\}$ and $B \sim A = \{9\}$

and $A \Delta B = \{2, 4, 9\}$

Equivalent Sets: Two finite sets A and B are said to be equivalent if their cardinal numbers are same , *i.e.*, $n(A) = n(B)$.

1.9.5 Law of Excluded Middle and Law of Contradiction

Two special properties of set operations are known as the excluded middle axioms and law of contradiction. The excluded middle axioms are very important because they are the only set operations described here that are not valid for both classical sets and fuzzy sets. Let A be any subset of universal set X. Then , we define.

(i) Axiom of the excluded middle: $A \cup A' = U$

(ii) Axiom of the contradiction: $A \cap A' = \phi$

Theorem 1.

(i) $A \cup \phi = A$ *(ii)* $A \cap \phi = \phi$ *(iii)* $A \cup A = A$

(iv) $A \cap A = A$ *(v)* $A \cup B = B \cup A$ *(vi)* $A \cap B = B \cap A$

Proof.

(i) Let x be an arbitrary element of $A \cup \phi$.

i.e., $x \in A \cup \phi$

have

$$4a - 2 = 2a \text{ and } b + 4 = 4$$

Therefore, $4a - 2a = 2 \Rightarrow a = 1$ and $b + 4 = 4 \Rightarrow b = 0$

Example 4. *If $A = \{1, 2, 3, 4\}$ and $B = \{4, 5\}$, represent $A \times B$, $B \times A$ and $B \times B$ pictorially and find their values.*

Solution. Given $A = \{1, 2, 3, 4\}$ and $B = \{4, 5\}$

$A \times B = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5), (4, 4), (4, 5)\}$

$B \times A = \{(4, 1), (5, 1), (4, 2), (5, 2), (4, 3), (5, 3), (4, 4), (5, 4)\}$

And $B \times B = \{(4, 4), (4, 5), (5, 4), (5, 5)\}$

Pictorially, $A \times B$, $B \times B$ and $B \times A$ can be represented as shown in figure 12.

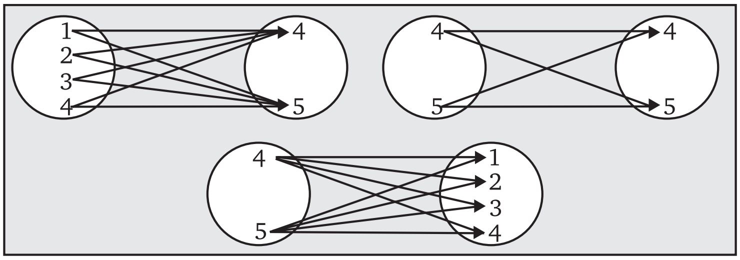

Fig. (12)

Example 5. *Let $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$. Determine (i) $A\times B$, (ii) $B\times A$. Also represent $A\times B$ and $B\times A$ graphically.*

Solution. (i) Given $A = \{1, 2, 3, 4\}$ and $B = \{5, 7, 9\}$. Then,

$A \times B = \{(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9)$ $(3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)\}$

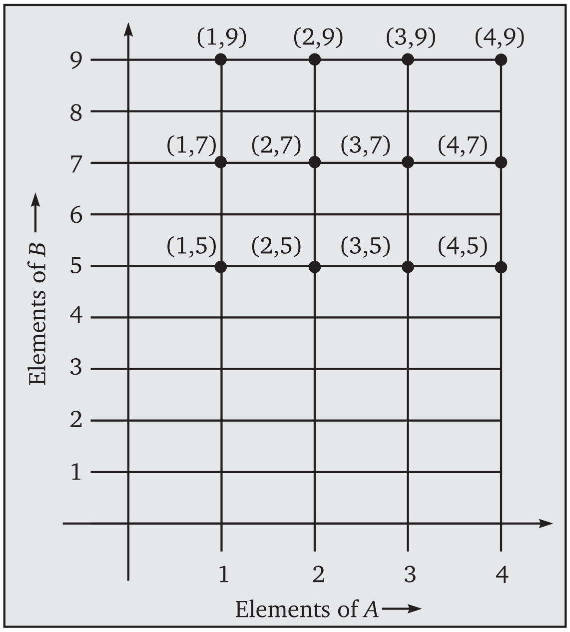

Fig. (13) : $A \times B$

(iii) We have $(a, b)\ R\ (c, d)$ iff $ad = bc\ \forall\ a, b, c, d \in \mathbf{N}$

Therefore, $(a, b)\ R\ (c, d), (c, d)\ R\ (e, f) \Rightarrow (a, b)\ R\ (e, f)\ \forall\ a, b, c, d \in \mathbf{N}$

Using $(a, d), (c, f) = (b, c)(d, e)$

$\Rightarrow \quad (a, f) = (b, e) \Rightarrow R$ is transitive

Hence, from (i), (ii) and (iii), we conclude that R is an equivalence relation.

Example 7. *Let R_1 and R_2 be two relations on a set A, where $A = \{1, 2, 3, 5\}$ such that*

$$R_1 = \{(1, 1), (1, 2), (1, 5), (2, 1), (2, 5)\}$$

and $$R_2 = \{(3, 3), (3, 2), (2, 3), (1, 2), (2, 1)\}$$

Then, which of the following statement is false :

(i) $R_1 \cup R_2$ *is symmetric* *(ii)* $R_1 \cap R_2$ *is transitive*

(iii) $R_1 \cap R_2$ *is symmetric* *(iv)* $R_1 \cup R_2$ *is transitive.*

Solution. (i) As $(1, 2) \in R_1$, also $(2, 1) \in R_1$, therefore, it is symmetric and as $(1, 2) \in R_2$, also $(2, 1) \in R_2 \Rightarrow R_2$ is symmetric.

Now, $R_1 \cup R_2 = \{(1, 1), (1, 2), (1, 5), (2, 1), (2, 5), (3, 3), (3, 2), (2, 3)\}$

In $R_1 \cup R_2$, as $(1, 2) \in R_1 \cup R_2$, also $(2, 1) \in R_1 \cup R_2 \Rightarrow R_1 \cup R_2$ is symmetric

Therefore, (i) is true.

(ii) We have $R_1 \cap R_2 = \{(1, 2), (2, 1)\}$

$\Rightarrow$ (1, 1) should also belong to $R_1 \cap R_2$.

But in this case $(1, 1) \notin R_1 \cap R_2$. Hence, $R_1 \cap R_2$ is not transitive.

Therefore, (ii) is false.

(iii) We have, $R_1 \cap R_2 = \{(1, 2), (2,1)\}$

$(1, 2) \in R_1 \cap R_2$ and also $(2, 1) \in R_1 \cap R_2$.

Therefore, (iii) is true.

(iv) In $R_1 \cup R_2$, $(1, 2) \in R_1 \cup R_2$

and $(2, 5) \in R_1 \cup R_2$, also $(1, 5) \in R_1 \cup R_2$

$\Rightarrow\ R_1 \cup R_2$ is transitive

Therefore, (iv) is true.

Example 8. *If A be the set of all triangles in a plane and $R = \{(a, b) : \Delta a = \Delta b\}$, i.e.,*

$aRb \Leftrightarrow$ Area of triangle a = Area of triangle b, then show that R is an equivalence relation.

Solution. (i) Since, for all $a \in A$ we have $\Delta a = \Delta a$

Therefore, $aRa \Rightarrow R$ is reflexive.

(ii) For any $a, b \in A$, we have $(a, b) \in R \quad \Rightarrow \Delta a = \Delta b$

$\Rightarrow \quad \Delta b = \Delta a \Rightarrow \quad (b, a) \in R$

Therefore, $(b, a) \in R$, i.e., $bRa \Rightarrow R$ is symmetric.

(iii) For all $a, b, c \in A$, we have $(a, b) \in R$, $(b, c) \in R$

$$\Delta a = \Delta b \text{ and } \Delta b = \Delta c \quad \Rightarrow \quad \Delta a = \Delta c \quad \Rightarrow \quad (a, c) \in R$$

Therefore, R is transitive.

Hence, from (i), (ii) and (iii), we conclude that R is an equivalence relation.