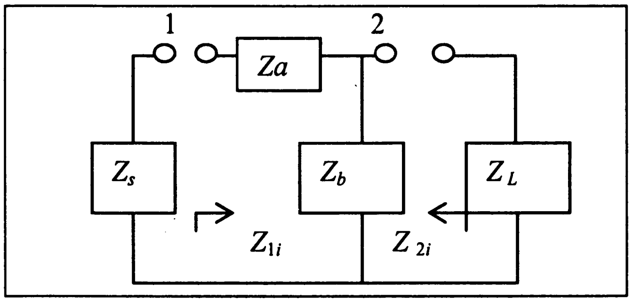
Impedance Matching

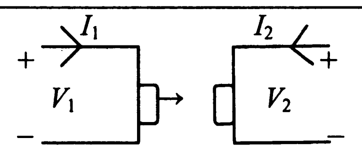


- L-section Z_a and Z_b is inserted between impedances Z_s (source) and Z_{1i} (load) so that the source sees an impedance Z_{1i} and seen an impedance Z_{2i}. Such an arrangement is called *impedance matching.*
- Impedance Z_{1i} and Z_{2i} is called **image impedance.**
- $Z_a^2 = Z_{1i}(Z_{1i} - Z_{2i})$

  $Z_a.Z_b = Z_{1i} - Z_{2i}$
- If $Z_{1i} < Z_{2i}$; Z_a and Z_b are reactive for purely resistive image impedances. Also one of the Z_a, Z_b is inductive and the other capacitive.

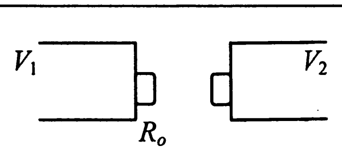
Impedance Matrix of Gyrator

- It is a device that gyrates the current of one port into a voltage at the other and vice versa.



$V_1 = -\alpha I_2$; $V_2 = \alpha I_1$

As impedance transforming element.



$V_1 + R_o I_2 = 0$

$V_2 = R_o I_1$

- Matrix form $\begin{vmatrix} 0 & -\alpha \\ \alpha & 0 \end{vmatrix}\begin{vmatrix} i_1 \\ i_2 \end{vmatrix} = \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}$
- If the gyrator is terminated in an impedance Z_L, the driving point impedance

$$Z_d = \frac{Z_{11}Z_L + \Delta_z}{Z_{22} + Z_L} \; ; \; Z_{in} = (\alpha^2/Z_L)$$

- If Z_L is capacitor, then $Z_L = (1/sC)$

  $Z_{in} = sC\alpha^2$ give inductor of value= $\alpha^2 C$
- If Z_L is an inductor $Z_L = sL$

  $Z_{in} = (\alpha^2/s_L)$ gives capacitor of value = (L/α^2)
- Used to simulate inductor from capacitor

Network Functions

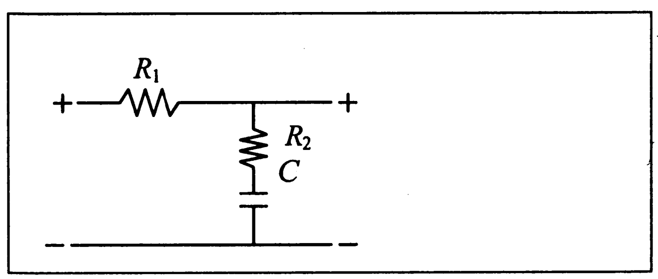
- A transfer function is the ratio of laplace transform of the output $Y(s)$ to laplace transform of the input $X(s)$.
- By setting the denominator of the transfer function to zero, and obtaining the roots by solving the equation, we can know the system's stability by considering by setting $s = 0$, i.e. $G(0)$ give DC gain where $G(s)$ is transfer function.

Procedure for Deriving Transfer Functions

Following assumption is made in deriving the transfer function.

- It is assumed that no loading of one system on other. If the system has more than one non loading element, the transfer function of each element can be determined independently and the overall transfer function can be obtain by multiplying the individual transfer function.
- If the system consisting of elements, which load each other, the overall transfer function should be derived by the basic analysis.

Lag Network



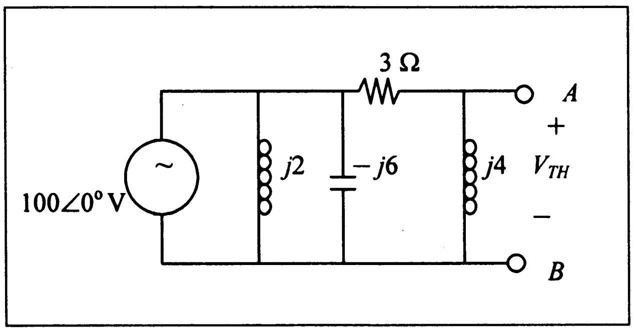
- Transfer function

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1 + sCR_2}{1 + sC(R_1 + R_2)} = \frac{1 + sT}{1 + s\alpha T}$$

Where, $T = R_2C$; $\alpha = \dfrac{R_1 + R_2}{R_2} : \alpha > 1$

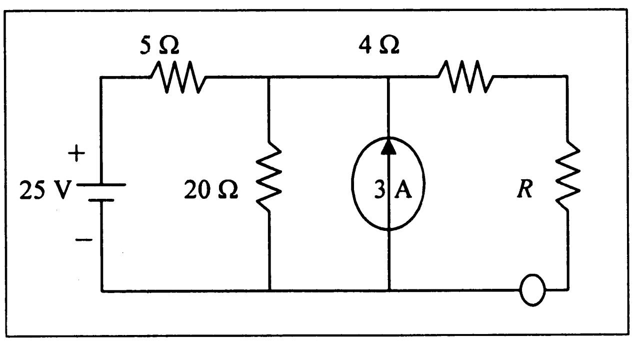
(a) − 1/2 (b) + 1/2
(c) − 3/2 (d) + 3/2

17. The Thevenin equivalent voltage V_{TH} appearing between the terminals A and B of the network shown in figure is given by



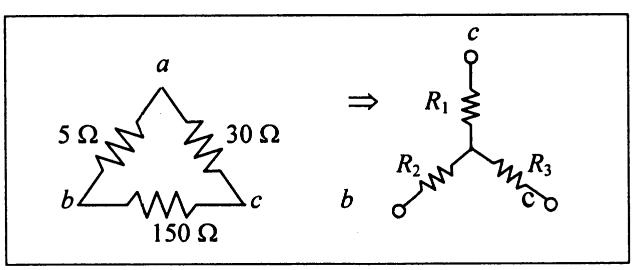
(a) $j16\,(3 - j4)$ (b) $j16\,(3 + j4)$
(c) $16\,(3 + j4)$ (d) $16\,(3 - j4)$

18. The value of R (in ohms) required for maximum power transfer in the network shown in figure is



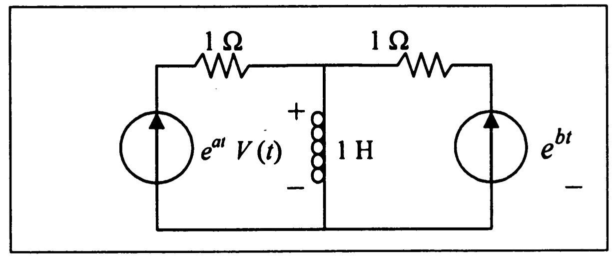
(a) 2 (b) 4
(c) 8 (d) 16

19. A Delta-connected network with its wave-equivalent is shown in figure. The resistances R_1, R_2, and R_3 (in ohms) are respectively



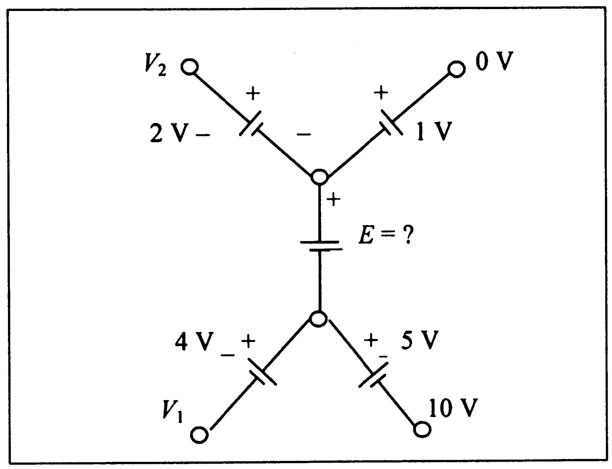
(a) 1.5, 3 and 9 (b) 3, 9 and 1.5
(c) 9, 3 and 1.5 (d) 3, 1.5 and 9

20. In the circuit of figure, the voltage $V(t)$ is



(a) $e^{at} - e^{bt}$ (b) $e^{at} + e^{bt}$
(c) $ae^{at} - be^{bt}$ (d) $ae^{at} + be^{bt}$

21. In the circuit of figure, the value of the voltage source E is



(a) − 16 V (b) 4 V
(c) − 6 V (d) 16 V

22. Given that

$$L[f(t)] = \frac{s+2}{s^2+1},\ L[g(t)] = \frac{s^2+1}{(s+3)(s+2)}$$

$$h(t) = \int_0^t f(T)\, g(t-T)\, dT$$

(a) $\frac{s^2+1}{s+3}$ (b) $\frac{1}{s+3}$

(c) $\frac{s^2+1+s+2}{(s+3)(s+7)}$ (d) None of the above

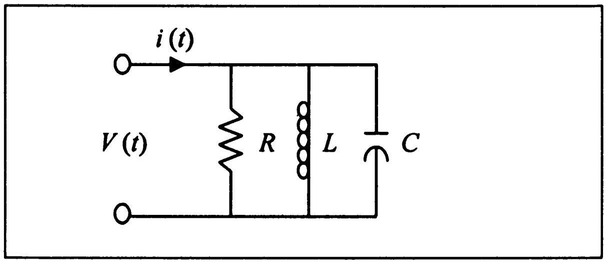
23. Linear time invariant system has an impulse response e^{2t}, $t > 0$. If the initial condition are zero and the input is e^{3t}, the output for $t > 0$ is

(a) $e^{3t} - e^{2t}$ (b) e^{5t}
(c) $e^{3t} + e^{2t}$ (d) None of the above

24. In figure, the steady state output voltage corresponding to the input voltage $3 + 4 \sin 100tV$ is

(a) $L_1 + L_2 + M$ (b) $L_1 + L_2 - M$
(c) $L_1 + L_2 + 2M$ (d) $L_1 + L_2 - 2M$

87. The circuit shown in figure with $R = (1/3)\ \Omega$, $L = (1/4)$ H, $C = 3$ F has input voltage $V(t) = \sin 2t$. The resulting current $i(t)$ is

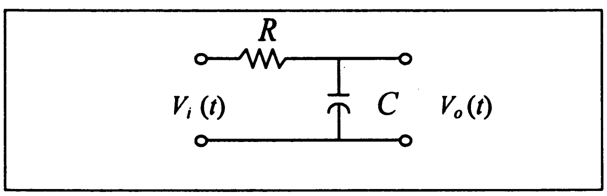


(a) 5 sin (2*t* + 53.1°)
(b) 5 sin (2*t* – 53.1°)
(c) 25 sin (2*t* + 53.1°)
(d) 25 sin (2*t* – 53.1°)

88. For the circuit shown in figure the time constant RC = 1 ms. The input voltage is

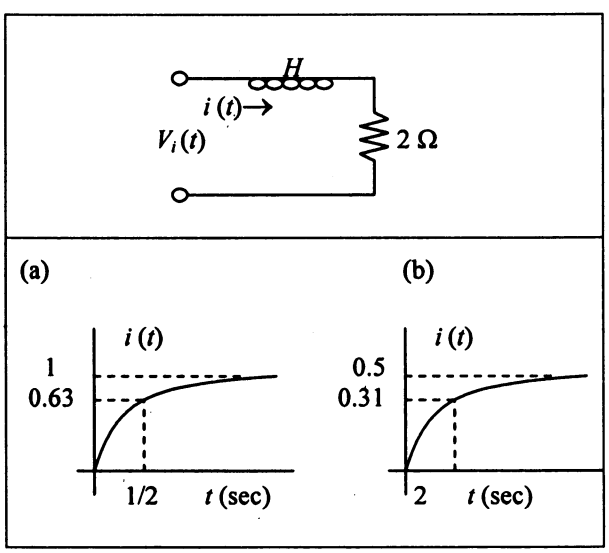
$$V_i(t) = \sqrt{2} \sin 10^3 t.$$

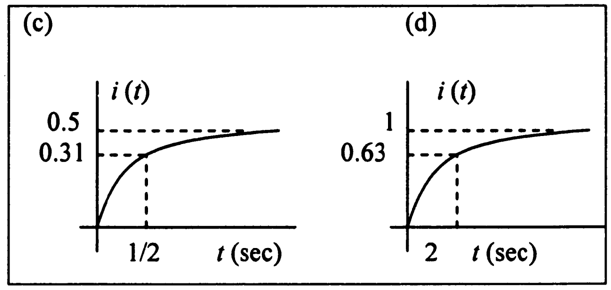
The output voltage $V_o(t)$ is equal to



(a) $\sin(10^3 t - 45°)$ (b) $\sin(10^3 t + 45°)$
(c) $\sin(10^3 - 53°)$ (d) $\sin(10^3 t - 53°)$

89. For the *R-L* circuit shown in figure the input voltage $v_i(t)$ = $u(t)$. The current $i(t)$ be





90. The *Z*-transform of the following real exponential sequence:

$$x(nT) = a^n \ ; \quad nT > 0$$
$$x(nT) = 0 \ ; \quad nT < 0, \eta > 0$$

is given by

(a) $1 - z^{-1} 4 \ ; |z| > a$

(b) $\dfrac{1}{1 - a z^{-1}} \ ; \ |z| > a$

(c) 1 for all z

(d) $1 - az^{-1} \ ; \ |z| > \eta$

Set 1 Answers Electric Circuits

1. **Ans.: (a)**
Solution: Maximum current flow at resonance, i.e. $Z_{in} = (V / R)$ at $\omega = \omega_o$

2. **Ans.: (d)**
Solution

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} A' & B' \\ C' & D' \end{vmatrix} \begin{vmatrix} A'' & B'' \\ C'' & D'' \end{vmatrix}$$

3. **Ans.: (c)**
Solution

$$H(s) = \frac{K(S-1-j)(S-1+j)(S-1)}{(S+1-j)(S+1+j)(S+1)}$$

$$= \frac{K\left(S^2 - 2S + 1\right)(S-1)}{\left(S^2 + 2S + 1\right)(S+1)}$$

$$= \frac{K(S-1)^2 (S-1)}{(S+1)^2 (S+1)} = \frac{K(S-1)^2}{(S+1)^2}$$

$$= \frac{K\left[\tan^{-1}(1/-1)\right]^2}{\left[\tan(1/1)\right]^2} = K\angle\text{constant angle}$$

All pass filter having constant magnitude.

4. **Ans.: (a)**
Solution: Real part of pole and zero must be negative or zero. Since *RC* network is used they must be simple.

$$= \int\left(\frac{V_R^2}{R}\right)dt = \int_0^{\infty} \frac{Vs^2\, e^{-2t/RC}}{R}\,dt$$

$$= \left(\frac{V_R^2}{S}\right)\int_0^{\infty} e^{-2t/RC}\,dt$$

$$= \int Vs^2\, e^{-2t/RC} \times (-)\frac{RC}{2}\Big|_0^{\infty}$$

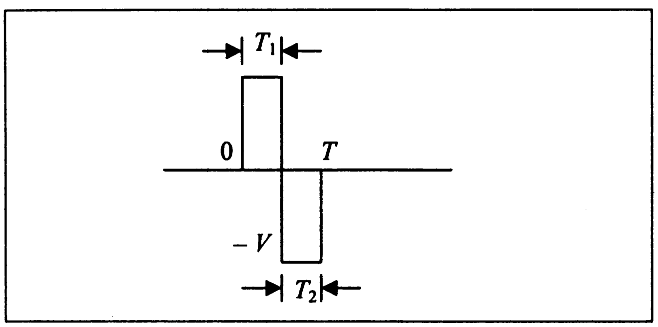
$$= V_S^2 \times RC \times 1 = 1$$
$$= (1/2)\, CV_S^2$$

We know that,
Energy stored in capacitor = $(1/2)\, CV_S^2$
Total energy supplied = $(1/2)\, CV_S^2 + (1/2)\, CV_S^2$
$= CV_S^2$

Hence, $\dfrac{\text{Total energy supplied}}{\text{Energy stoted in capacitor}} = 0.5$

53. Ans.: (d)
Solution: The equivalent inductance for L_1 and L_2 in series and mutual inductance M is added than result is given by the formula $L = L_1 + L_2 + 2\text{ M}$
Where, $M = K\sqrt{(L_1L_2)}$
Given, $L_1 = L_2 = 2$ H
$K = 0.1$
$\therefore$ Total inductance $(L) = 2 + 2 + 2\ 0.1\ \sqrt{(2 \times 2)}$
$= 4 + 0.4 = \mathbf{4.4\ H}$

54. Ans.: (a)
Solution



$T_1 + T_2 = T$

Mean square value $= \dfrac{1}{T}\left(\int_0^{T_1} V^2\, dt + \int_{T_1}^{T} V^2\, dt\right)$

$$= \frac{1}{T}\left(V^2T_1 + V^2T - V_2T_1\right) = V^2$$

$\therefore$ RMS value = $\sqrt{(V^2)}$ = **V**

55. Ans.: (b)
Solution: $F(s) = \dfrac{2(s+1)}{(s^2+2s+5)} = \dfrac{2(s+1)}{(s^2+1)^2+4}$

$$= 2\left(\frac{s+1}{(s+1)^2+4}\right)$$

This is standard lap lace transform for the function

$$LT\ |\exp(-a\,t)|\ \cos\omega t = \frac{(s+a)}{(s+a)^2+\omega^2}$$

Where, $a = 1,\ \omega = 2$
Hence, $f(t) = 2e^{-t}\cos 2t$
$f(0) = f(0+) = 2e^{-0}\cos 2 \times 0 = \mathbf{2}$
$f(\infty) = 2e^{-\infty} = \mathbf{0}$

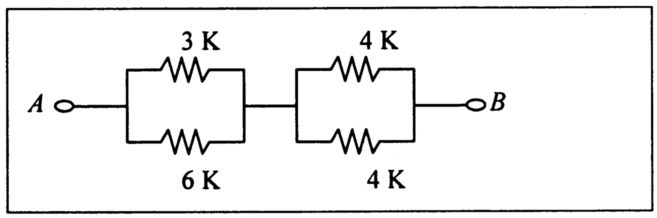
or

$$f(0) = \underset{s\to\infty}{Lt}\ sF(s) = \frac{s(2)(s+1)}{s^2+2s+5}$$

$$= \frac{2s^2+s}{s^2+2s+5} = \frac{2+(1/s)}{1+(2/s)+(5/s^2)}$$

$f(0) = \mathbf{2}$

56. Ans.: (a)
Solution: The circuit is drawn below by shorting the voltage source and opening the resistor R, we fined the Z_{eq}.



Taking the Thevenin's equivalent between points A, B

$$Z_{eq} = \left(\frac{6\times3}{6+3} + \frac{4\times4}{4+4}\right) = (18/9) + (16/8)$$

$= 4\ \Omega$

Load impedance should be equal to the complex conjugate of source impedance. When the above circuit is represented by thevinin's equivalent circuit.

For maximum power transfer $R = R_{eq} = 4\ \text{k}\Omega$

57. Ans.: (c)

Solution: $i = 3 + 4\sin(100t + 45°) + 4\sin(300t + 60°)$ amp

$$I\text{ (RMS value)} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = 5\text{ amp}$$

Power = i^2 (RMS) $R = 25 \times 10 = \mathbf{250\ W}$

58. Ans.: (a)
Solution