*Example:* $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

It is upper triangular matrix be cause all elements below the diagonal are zero.

b) ***Lower triangular matrix:*** A square matrix is called lower triangular matrix if all the non diagonal elements above the diagonal are zero.

*Example:*

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & -1 \end{pmatrix}$$

Here all elements above the diagonal are zero.
A is called lower triangular matrix .

**Equal matrix:** Two matrices are said to be equal if they are of the same order and each corresponding element of A are equal to B.

*Example:* $\begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2 \times 2}$ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}_{2 \times 2}$

if a = 1, b = 2, c = 3, d = 4

A joint of a square matrix A is the transpose of cofactors matrix A.

**Note:** $(AB)^1 = B^1 A^1$

***Inverse of matrix:*** Inverse of a square matrix. A is written as $A^{-1}$ and obtained by using the formula $A^{-1} = \frac{1}{|A|}$. Adj A, where $|A| \neq 0$.

***Echelon form of a matrix:*** If A be the given matrix of order m x n than it can be reduced to echelon form by row transformations if we can make all the entries equal to zero . Then R (A) = Number of non zero rows.

***Definition:*** A determinant is a function depending on n that associates a scalar det (A) to every $(n \times n)$ square matrices A deter,omamt of nth order is consist of n rows and n columns and is of the form:

$$\begin{vmatrix} a_{11}a_{12}\cdots a_{1n} \\ a_{21}a_{22}\cdots a_{2n} \\ \cdots\cdots\cdots\cdots \\ a_{n1}a_{n2}\cdots a_{nn} \end{vmatrix}$$

This determinant is the elimination of variables $X_1, X_2, ..X_n$ from the equations.

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = 0$

$a_{21}x_1 + a_{22}x_2 + ..a_{2n}x_n = 0$

$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n, = 0$

***Consistency and so/ⁿ of algebraic equations***

In argument matrix A* we reduce echelon for if

a) Rank of A* = Rank of A then equation are consistent.

b) If Rank of A* $\neq$ Rank of A then the system is inconsistent. (i.e. no solution).

In case of homogenous system of equations A x = 0

a) If rank = Number of unknowns then it has zero solution.

b) If rank < Number of unknowns then it has infinitely many solution.

***Definition of rank:*** Let A be a matrix of order m x n : than r is the rank of the matrix if all (r + 1 ) th order minors are zero an at least one minor of order r is non zero.

*Note:*

1. Rank of a matrix is written as ρ.
2. Rank of a Null matrix is zero.
3. Rank of a square matrix which is non singular if order n is n.
4. Rank of matrix A of order m × n ≤ min of {m.n}.

***Definition of Normal Form***

A matrix is said to be in Normal form if it can be expressed.

$$\begin{pmatrix} In-r & 0 \\ 0 & 0 \end{pmatrix}$$

Where In – r is Identity matrix of order (n – r)

***Hermitian matrix:*** A matrix is called her Hermitian if fA* = A or $A^\theta = A$. Where A* means ( $A^1$).

*Note:*

Matrix a must be square matrix. Each diagonal entry of skew Hermitian matrix is either purely imaginary or zero.

***Unitary Matrix***

A matrix is called unitary if A* A = I

***Similar Matrices***

A matrix B is said to be similar to another matrix a if ∃. An in veritable matrix P such that AP = PB or A = $PBP^{-1}$ or B = $p^{-1}$ AB. Note. A, B, P are square matrix of same order over a field F.

*Note:*

A and B have same eigen values. Characteristic equation Eigen values and Eigen vectors. Cayley hamilton the order:

(a) $|X(\omega)|\cdot|\omega|$

(b) $|X(\omega)|\cdot\omega$

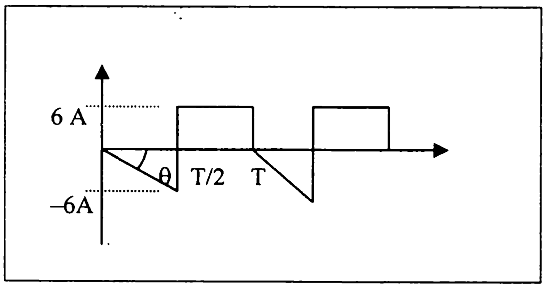
(c) $\omega^2\cdot|X(\omega)|$

(d) $|\omega|\cdot|X(\omega)|\cdot e^{-j\omega t_d}$

## Answers
## Signals and Systems

**1. Ans. (a)**
**Solution:**
Straight line equation, y = mt + C
When m = slop of line = tan θ.

then, $m=\frac{-6}{T/2}=\frac{-12}{T}$ and C = 0




$f(t) = mt \quad ; 0\le t\le T/2$
$= 6 \quad ; T/2\le t\le T$

rms value for any waveform $=\sqrt{\frac{1}{T}\int_o^T f^2(t)\cdot dt}$

$$=\left[\frac{1}{T}\left(\int_o^{T/2}(mt)^2\cdot dt+\int_{T/2}^{T}6^2\cdot dt\right)\right]^{1/2}$$

$$=\left[\frac{1}{T}\left(\frac{144}{T^2}\times\frac{T^3}{3\times 8}+36\times\frac{T}{2}\right)\right]^{1/2}$$

$$=[6+18]^{1/2}=\sqrt{24}=2\sqrt{6}$$

**2. Ans. (a)**
**Solution:**

$$V(t)=\frac{2t}{T},\ 0\le t\le T/2$$
$$=0;T/2\le t\le T$$
$$V(t)_{rms}=\sqrt{\frac{1}{T}\int_o^T|V(t)|^2dt}$$
$$=\sqrt{\frac{1}{T}\int_o^{T/2}\frac{4}{T^2}t^2dt}$$
$$=\sqrt{\frac{1}{T}\cdot\frac{4}{T^2}\frac{t^3}{3}\Big|_o^{T/2}}=\sqrt{\frac{4}{T^3}\cdot\frac{T^3}{8\times 3}}=\sqrt{\frac{1}{6}}$$

**3. Ans. (a)**
**Solution:**
Apply find value theorem
$$x(\infty)=\lim_{t\to\infty}f(t)=\lim_{s\to 0}sF(s)$$
$$=\lim_{s\to 0}\frac{s(5s^2+23s+6)}{(s^2+2s+2)s}=3$$

**4. Ans. (b)**
**Solution:**
$$F(z)=\frac{1}{z+1}=\frac{z+1-z}{z+1}$$
$$=1-\frac{z}{z-(-1)}$$
$$\therefore\ ZT^{-1}[F(z)]=\delta(t)-(-1)^n$$
$$\left[\therefore ZT^{-1}\left(\frac{z}{z-a}\right)=a^n\right]$$

**5. Ans. (d)**
**Solution:**
$f(x)=\sin^2x$
Since, $f(x)$ is an even function, hence it has no sine terms.
$$a_o=\frac{2}{T}\int_o^T f(x)dx$$
$$a_0=\frac{2}{2\pi}\int_o^{2T}f(x)dx=1$$
$$\therefore\ f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos nx$$
f(x) = 0.5 + terms containing cosine

**6. Ans. (d)**
**Solution:** $y(t)=e^{-|x(t)|}$

Therefore, $e^{-x}$ is always convergent even when x is not bounded

Therefore $e^{-x}$ is bounded even though $x^2$ is not bounded

**7. Ans. (a)**
**Solution:**
Given data input and output Sequences are
$$x[n]=\{-1,\underset{\uparrow}{2}\},-1\le n\le 0$$
$$y[n]=\{-1,\underset{\uparrow}{3},-2\},-1\le n\le 2$$
If impulse response of system is $h[n]$ then output
$$y[n]=h[n]*x[n]$$
Since length of convolution $(y[n])$ is $-1$ to 2, $x[n]$ is of length – to to 0

**Common data for Q. 3 and Q. 4**

**3.** A signal is processed by a causal filter with transfer function $G(s)$. For a distortion free output signal waveform $G(s)$ must.

(a) Provide zero phase shift for all frequency
(b) Provide constant phase shift for all frequency
(c) Provide linear shift that is proportional to frequency
(d) Provide a phase shift that is inversely properly proportional to frequency

**4.** $G(z) = \alpha z^{-1} + \beta z^{-3}$ is a low-pass digital filter with a phase characteristics same as that of the above question if.

(a) $\alpha = \beta$  (b) $\alpha = -\beta$
(c) $\alpha = \beta^{1/3}$  (d) $\alpha = \beta^{-(1/3)}$

## Answers
## Signals and Systems

**1. Ans. (c)**
**Solution:**
For RC low pass filter

$$H(f) = \frac{1}{j2\pi fRC + 1}$$

$$H(o) = 1$$

Given $\frac{H(f_1)}{H(o)} \geq 0.95$

$$\frac{1}{\sqrt{4\pi^2 f_1^2 R^2 C^2 + 1}} \geq 0.95$$

$$\Rightarrow \frac{1}{4\pi^2 R^2 C^2 f_1^2 + 1} > 0.9025$$

$$4\pi^2 R^2 C^2 f_1^2 \leq 0.108$$

$$f_1^2 < \frac{0.108}{4\pi^2 R^2 C^2}$$

$$f_1 < \frac{0.329}{2\pi RC}\ f_1 < \frac{0.329}{2\pi RC} < \frac{0.329}{2\pi \times 10^3 \times 10^{-6}}$$

$$= 52.2$$

**2. Ans. (d)**
**Solution:**

$$H(f) = \frac{1}{j2\pi fRC + 1}$$

$$\phi(f) = -\tan^{-1}(2\pi fRC),$$

Group delay $\tau_g = -\frac{d\phi(f)}{d(f)}$

$$= -\frac{1}{(2\pi fRC)^2 + 1} \times 2\pi RC$$

$f = 1000\ Hz$, $R = 1.0\ k\Omega$, C

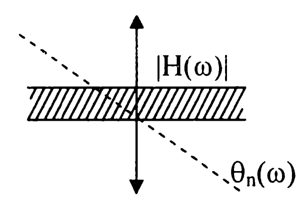
$C = 1\ \mu F$ Putting f, R and C in this equation,

Group delay $\tau_g =$

$$\frac{2 \times 3.14 \times 1 \times 10^3 \times 10^{-6}}{4 \times 3.14 \times 3.14 \times 100 \times 100 \times 1 \times 10^3 \times 10^3 \times 10^{-6} \times 10^{-6} + 1}$$

$$= \frac{6.28}{1.39} = 4.51\ ms$$

**3. Ans. (c)**
**Solution:**




For Distortion Less filter
$Y(j\omega) = A(j\omega) \cdot X(j\omega)$
$H(j\omega) = A e^{-j\omega}$
Magnitude → constant
Phase → linear

**4. Ans. (d)**
**Solution:**

$$G(z)\big|_{z=e^{j\omega}} = \alpha e^{-j\omega} + \beta e^{-3j\omega}$$

For linear phase characteristic $\alpha = \beta$.

etc.) is controlled according to a preset program are often used in such system.

### 7.3.2 CHARACTERISTICS OF FEEDBACK

The presence of feedback typically imparts the following properties to a system :

1. Increased accuracy. For example, the ability to faithfully reproduce the input. This property is illustrated through out the text.
2. Tendency towards oscillation or instability.
3. Reduced sensitivity of the ratio of output to input to variations in system parameters and other characteristics.
4. Reduced effects of non-linearities.
5. Reduced effects of external disturbances or noise.
6. Increased bandwidth. The bandwidth of a system is a frequency response measure of how well the system responds to (or filters) variations (or frequencies) in the input signal.

### 7.3.3 STABILITY

Stability in a system implies that small changes in the system input in initial conditions or in system parameters do not result in large changes in system output.

A linear time-invariant system is stable if the following notions of system stability are satisfied:

1. When the system is excited by a bounded input the output is bounded.
2. In the absence of the input, the output tends towards zero, irrespective of initial condition. This is known as *asymptotic stability*.

#### General Conclusions Regarding System Stability

1. If all the roots of the characteristic equation have negative real part, then the impulse response is bounded and eventually decreases to zero. The system is bounded-input, bounded-output *stable.*
2. If any root of characteristic equation has a positive part, the output is unbounded, so system is unstable.
3. If the characteristic equation has repeated roots on the jù axis, output is unbounded and therefore, system is *unstable.*
4. If the first condition is satisfied except for the presence of one or more non-repeated roots on the jù axis, the system is *limitedly stable or marginally stable.*

#### Type of Stability

- *Absolutely Stable* with respect to a parameter of the system if it is stable for all values of this parameter.
- *Conditionally stable* with respect to a parameter, if the system is stable for only certain bonded ranges of value of this parameter.

### 7.3.4 SENSITIVITY

Feedback reduces the error. It reduce the sensitivity of the system due to parameter variation and unwanted internal and external disturbances. The parameters of a system have tendency to vary under a variety of changing condition, these parameters variation have adverse effect on system performance.

### 7.3.5 REDUCTION OF PARAMETER VARIATION BY USE OF FEEDBACK

The beneficial effects of feedback in feedback system with high loop gain, which will be elaborated in this section, are enumerated below.

One of the primary purpose of using feedback in control systems is to reduce the sensitivity of the system to parameter variations. The parameters of a system may vary with age, with changing environment (e.g. ambient temperature), etc. Conceptually, sensitivity is a measure of the effectiveness of feedback in reducing the influence of these variations on system performance.

Let us define sensitivity on a quantitative basis. In the open-loop case

$$C(s) = G(s)\,R(s)$$

Suppose due to parameter variation $G(s)$ changes to $[G(s)+\Delta G(s)]$ where $|G(s)| >> |\Delta G(s)|$. The output of the open-loop system then changes to

$$C(s)+\Delta C(s) = [G(s)+\Delta G(s)]\,R(s)$$

$$\Delta C(s) = \Delta G(s)\,R(s) \qquad ...(1)$$

or

Similarly, in the closed-loop case, the output

$$C(s) = \frac{G(s)}{1+G(s)H(s)}R(s)$$

changes to

$$C(s)+\Delta C(s) = \frac{G(s)+\Delta G(s)}{1+G(s)H(s)+\Delta G(s)H(s)}R(s)$$

due to the variation $\Delta G(s)$ in $G(s)$ the forward path transfer function.

Since $|G(s) >> |\Delta G(s)$, we have from the above, the variation in the output as

$$\Delta C(s) \approx \frac{\Delta G(s)}{1+G(s)H(s)}R(s) \qquad ...(2)$$

From Eqns. (1) and (2) it is seen that in comparison to the open-loop system, the change in the output of the closed-loop system due to variation $G(s)$ is reduced by a factor of $[1 + G(s)H(s)]$ which is much greater than unity in most practical cases over the frequency $(s = j\omega)$ of interest.

### 7.3.6 SENSITIVITY OF CONTROL SYSTEM

The term *system sensitivity* is used to describe the relative