$$= LT^{-1}\left[\frac{8(-4+1)}{(-4+2)(s+4)}+\frac{8(-2+1)}{(-2+4)(s+2)}\right]$$

$$= LT^{-1}\left[\frac{8(-3)}{(-2)(s+4)}+\frac{8(-1)}{(+2)(s+2)}\right]$$

$$= LT^{-1}\left[\frac{12}{(s+4)}-\frac{4}{(s+4)}\right]$$

$$= [12\,e^{-4t} - 4\,e^{-2t}]\,u(t)$$

16. Ans. (d)

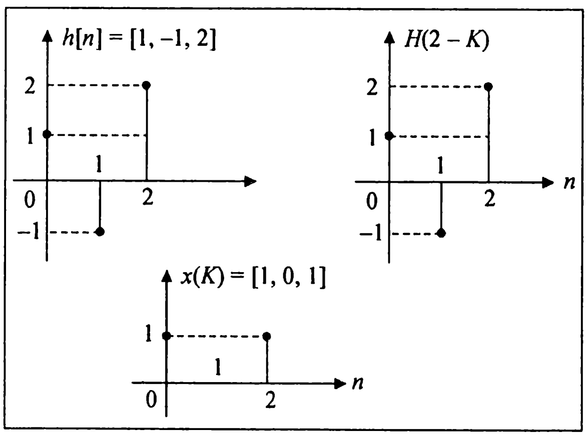
Solution: Use convolution result to get the total number of terms

$$h[n] = [1, -1, 2]$$

$$x[n] = [1, 0, 1]$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x(K)\,h[n-K]$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(K)\,h(2-K)$$



$$y(2) = 2 \times 1 + (-1) \times 0 + 1 \times 1 = 3$$

$$y(-1) = 0, y(0) = 1, y(1) = -1, y(2) = 3,$$

$$y(3) = -1, y(4) = 2, y(5) = 0$$

Hence, five non-zero samples.

17. Ans. (a)

Solution: R_1: $y(t) = t^2x(t)$

Linear time variant

$$R_2 : y(t) = t|\,x(t)\,|$$

Non, linear non-time invariant

$$R_3 : y(t) = |\,x(t)\,|$$

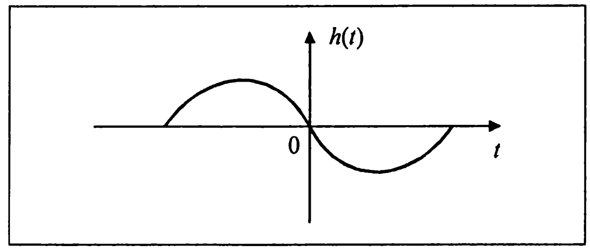
Non linear time invariant

$$R_4 : y(t) = x(t-5)$$

Liner time invariant

18. Ans. (b)

Solution:



$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\,h(2\tau)d\tau$$

As, $\lim\limits_{t\to\infty} h(t)$ or $\int_{-\infty}^{\infty} h(t)dt$ is finite for the given $h(t)$

So system is Bounded

So answer can be (b) or (c)

Now system is not causal $h(t)$ is defined for $t < 0$

so correct answer answer is (b)

19. Ans. (c)

Solution: Given that:

$$H(z) = \frac{2-\frac{3}{4}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$H(z) = \frac{\left(1-\frac{1}{4}z^{-1}\right)+\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

By partial fraction

$$H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)}+\frac{1}{\left(1-\frac{1}{4}z^{-1}\right)}$$

S_1: System will be causal for $|z| > \frac{1}{2}$; $|z| = \infty$

S_1: System will be stable for $|z| > \frac{1}{4}$; $|z| = 1$

System will be neither causal nor stable for ROC

$$\frac{1}{4} < |z| < \frac{1}{2}$$

S_1 true, S_2 false (System is stable nor causal), S_3 true.

$$=\left(\frac{1-e^{-sT}}{s}\right)\sum_{k=0}^{\infty} i(kT)e^{-skT}$$

$$L^{-1}\left[\sum_{k=0}^{\infty} i(kT)e^{-skT}\right]$$

$$=\sum_{k=0}^{\infty} i(kT)\ \delta(t-kT)=i(t)\ \delta_T(t) ..(2)$$

We can give a new input-output interpretation to above equation. Taking the inverse Laplace transform of the infinite sum.

$$L^{-1}\left[\sum_{k=0}^{\infty} i(kT)e^{-skT}\right]$$

$$=\sum_{k=0}^{\infty} i(kT)\ \delta(t-kT)=i(t)\ \delta_T(t) ...(3)$$

Thus the output $o(t)$ of pulse sampler and ZOH can be produced by impulse sampled $i(t)$ when pass through a transfer function.

$$G_o(s)=\frac{1-e^{-sT}}{s}. \qquad ...(4)$$

35. Ans. (a)

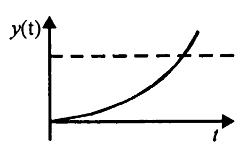
Solution:

$$100\frac{d^2Y}{dt^2}-20\frac{dy}{dt}+y=x(t)$$

$$100s^2Y(s)-20\,sY(s)+Y(s)=X(s)$$

$$\left(100s^2-20\,sY+1\right)Y(s)=X(s)$$

$$\frac{Y(s)}{X(s)}=\frac{1}{100\,s^2-20\,s+1}$$



$$=\frac{1}{100\left(s^2-\frac{1}{5}s+\frac{1}{100}\right)}$$

$$\omega_n=\frac{1}{10}$$

$$\delta_1=-1\,;\ 2\delta_1\omega_n=\frac{-1}{5}$$

Initially relaxed

$$Y'(0)=0$$

$$Y''(0)=0$$

Second Method

$$\frac{100d^2y}{dt^2}-\frac{20dy}{dt}+y=x(t)$$

Take Laplace transform both sides

$$(100s^2-20s+1)Y(s)=\frac{1}{s}$$

$$\left[\because x(t)=u(t)\Rightarrow X(s)=\frac{1}{s}\right]$$

$$Y(s)=\frac{1}{s(100s^2-20s+1)}$$

So we have poles with positive real part $\Rightarrow$ system is unstable.

36. Ans. (d)

Solution:

$$x(t)=e^{-2t}+\delta(t-6)$$

$$h(t)=u(t)$$

$$X(s)=\frac{1}{s+2}+e^{-6s}\ \text{and}\ H(s)=\frac{1}{s}$$

$$Y(s)=H(s)X(s)$$

$$=\frac{1}{s}\left[\frac{1}{s+2}+e^{-6s}\right]$$

$$=\frac{1}{s(s+2)}+\frac{e^{-6s}}{s}$$

$$=\frac{1}{2}\frac{1}{s}-\frac{1}{2(s+2)}+\frac{e^{-6s}}{s}$$

or, $y(t)=0.5(1-e^{-2t})u(t)+u(t-6)$

37. Ans. (b)

Solution:

$$s^2Y(s)+4s\,Y(s)+3Y(s)$$

$$=2s\,X(s)+4\,X(s)$$

$$Y(s)\left[s^2+4s+3\right]=X(s)[2s+4]$$

$$\frac{Y(s)}{X(s)}=\frac{2s+4}{s^2+4s+3}=\frac{2(s+2)}{(s+1)(s+3)}$$

if $\quad x(t)=e^{-2t}u(t)$

$$X(s)=\frac{1}{s+2}$$

$$y(n) = h(n)$$

$$y(n) = x(n) - \frac{1}{3}y(n-1)$$

$$Y(z) = X(z) - \frac{1}{3}Y(z)z^{-1}$$

$$Y(z) + \frac{1}{3}Y(z)z^{-1} \Rightarrow 1$$

$$Y(z) = \frac{1}{1+\frac{1}{3}z^{1}}$$

$$Y(z) = \frac{1}{1-\left(\frac{-1}{3}\right)z^{-1}}$$

$$y(n) = \left(\frac{-1}{3}\right)^{n} u(n)$$

74. Ans. (b)

Solution:

If $H(z)$ is discrete rational transfer function then,

where $\frac{Y[z]}{X[z]} = \frac{\text{zero}}{\text{pole}}$

For $H(z)$ to be stable the poles most be in unit circle

$$[H(z)]^{-1} = \frac{X(z)}{Y(z)}$$

The zeroes must be inside the circle.

So poles and zeroes must be inside the unit circle

75. Ans. (c)

Solution:

$$y(t) = \int_{-\infty}^{5t} x(\tau)d\tau$$

for $t = 1$

$$y(1) = \int_{-\infty}^{5} x(\tau)d\tau$$

Here output depends on future values of $x(t)$ as even though $t = 1$ output takes upto 5 times unit so system is non-causal.

Now if shift in input $x(t-t_o)$, then output is

$$y'(t) = \int_{-\infty}^{5t} x(\tau - t_o)d\tau$$

$$= \int_{-\infty}^{5t} x(\tau - t_o)d\tau$$

$$\tau - t_o = \tau'$$

or $$y'(t) = \int_{-\infty}^{(5t-t_o)} x(\tau')d\tau'$$

and $$y(t-t_o) = \int_{-\infty}^{(5t-t_o)} x(\tau)d\tau \neq y'(t)$$

So system is time variant.

76. Ans. (c)

Solution: $y(t) = \frac{d}{dt}\left[e^{-t}x(t)\right]$

$$y(t) = e^{-t}x'(t) + x(t)e^{-t}(-1)$$

$$y(t) = e^{-t}x'(t) - e^{-t}x(t)$$

$$y(t) = e^{-t}\left[x'(t) - x(t)\right]$$

At $t \to \infty$; $y(t) = 0$

So it's o/p is stable, because it is bounded even for $t \to \infty$.

77. Ans. (a)

Solution:

Characteristic of thermometer

$$\frac{2dT_i}{dt} = T_a - T_i$$

Take the Laplace transform

or, $$2sT_i(s) = T_a(s) - T_i(s)$$

$$2sT_i(s) + T_i(s) = T_a(s)$$

or, $$\frac{T_i(s)}{T_o(s)} = \frac{1}{2s+1} = \frac{0.5}{s+0.5}$$

$\Rightarrow$ $$0.5\,e^{-0.5t}$$

Hence, $$\omega = 0.5$$

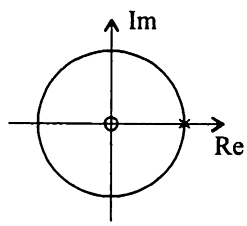
$$f = \frac{\omega}{2\pi} = \frac{1}{4\pi}Hz$$

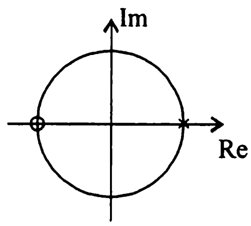
78. Ans. (b)

Solution:

$$x[n] = \left[\frac{1}{2}\right]^{n} u[n]$$

$$X[z] = \frac{z}{(z-\frac{1}{2})}$$





Given $h[2]=1$, $h[3]=-1$ and $h[k]=0$, otherwise

$h(k)$ can be represented as

$$h(k)=\delta[n-2]-\delta[n-3]$$

Taking z-transform,

$$H(z)=z^{-2}-z^{-3}=\frac{z^{-1}}{z^{3}}$$

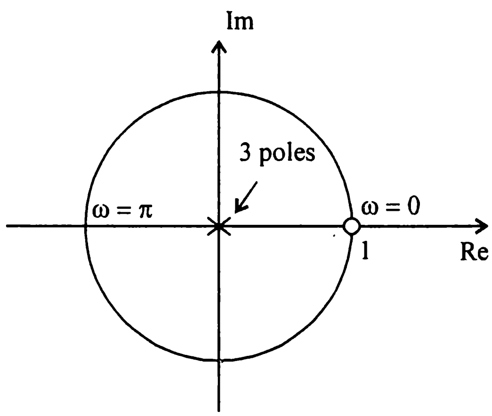


Figure: Pole-zero diagram of $\frac{(z-1)}{z}$

The given system does not represent a low pass filter because as it is not similar to a low pass filter.

* An FIR filter is one which has non-zero impulse response only for finite duration. It has all the poles located at $z=0$.

As given impulse response exists only for $k=2$ and $k=3$, therefore, given response represents a FIR filter.

20. Ans. (b)

Solution: $x(n) = \sin(\omega_o n + \phi)$

$$X(_n - n_o) = \sin[\omega_o (n - n_o) + \phi]$$

$$= \sin[\omega_o n - \omega_o n_o + \phi]$$

$$y(n) = Ax(n - n_o)$$

$$Y(e^{j\omega}) = A \exp[-j\omega n_o] X(e^{j\omega})$$

$$H(e^{j\omega}) = [Y(e^{j\omega}) / X(e^{j\omega})]$$

$$= A \exp[-j\omega n_o] = |H(e^{j\omega})| \angle H(e^{j\omega})$$

$$\angle H(e^{j\omega}) = -\omega n_o$$

$$\angle H(e^{j\omega}) = -\omega_o n_o + 2\pi k$$

General solution; k is a arbitrary integer

21. Ans. (a)

Solution:

$$G(z)=\alpha z^{-1}+\beta z^{-3}$$

$$G(z)\Big|_{z=e^{j\omega}} = \alpha e^{-j\omega}+\beta e^{-3j\omega}$$

Let $\alpha=\beta$, then $G(z)=\alpha z^{-j2\omega}\left(e^{j\omega}+e^{-j\omega}\right)$

$$=2\alpha e^{-j2\omega}\cos\omega$$

$\left|G\left(e^{j\omega}\right)\right|=\left|2\alpha\cos\omega\right|$ and $\theta(\omega)=-2\omega$

$\Rightarrow \frac{\theta(\omega)}{\omega}=2=$ constant

$\Rightarrow \theta(\omega)$ is proportional to ω.

22. Ans. (d)

Solution: The filter transfer function $T(s)$ can be written as the ratio of two polynomials as

$$T(s)=\frac{a_M s^M + a_{M-1}s^M + \cdots + a_0}{s^N + a_{N-1}s^N + \cdots + b_0}$$

The degree of the denominator, N, is the filter order. For the filter circuit to be stable, the degree of the numerator must be less than or equal to that of the denominator: $M \le N$. The numerator and denominator coefficients, a_0, a_1, ... a_M and b_0, b_1, ... a_{N-1} are real numbers. The polynomials in the numerator and denominator can be factored, and $T(s)$ can be expressed in the form

$$T(s)=\frac{a_M(s-z_1)((s-z_2))\cdots(s-z_M)}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

The numerator roots, z_1, z_2, ..., z_M, are the transfer-function zeros, or **transmission zeros**; and the denominator roots, $p_1, p_2, \ldots, p_N$, are the transfer function poles, or the natural poles. Each transmission zero or pole can be either a real or a complex number. Complex zeros and poles, however, must occur in conjugate pairs. Thus, if $-1+j2$ happens to be a zero, then $-1+j2$ also must be a zero

Write equation of all filter

$$T(s)=\frac{a_m s^m + a_{m-1}s^{m-1} + \ldots a_o}{b_n s^n + a_{n-1}s^{n-1} + \ldots b_o}$$

The degree of denominator N, is the filter order for the circuit to be stable.

If the transmission zero are located on the $j\omega$ axis, at the complex-conjugate locations $\pm j\omega_n$, then the mag