$$= \frac{1}{0.013} (0.0001)^{½} \frac{(0.645)}{(1.927)}$$

For α = 1.1 Radians Q = 0.257 m^3

$$\text{next approximation - } \alpha^{k+1} = 1.1 - \frac{f(\alpha^k)}{f'(\alpha^k)}$$

$$= 1.1 - \frac{(\pi + 2(1.1))(3+5\cos 2\alpha) - 2\sin\alpha}{(6 - 10\pi \sin 2\alpha - 10\alpha \sin 2\alpha + 10 \cos 2\alpha - 2\cos\alpha)}$$

= 1.06

$$Q = \frac{1}{0.013} \left(0.0001 \right)^{½} \frac{\left(\frac{\pi}{8} + \frac{1}{4} \left(1.06 + \frac{\sin(60.73 \times 2)}{2} \right) \right)^{5/3}}{\left(\frac{¶}{2} + 1.06 \right)}$$

$$= \frac{1}{0.013\ 2.} (0.01) \frac{(0.764)}{63} = 0.223\ m^3$$

Example : Minimum cost aggregate mix

A contractor has to supply the material for which he has two alternatives.

1. He takes out the material from first pit and supplies, for which he has to bear Rs 100/m^3.
2. Material from pit 2, for which he has to spend Rs 150/m^3 for loading, delivering and unloading.

The minimum quantity to be delivered is 1000 m^3. The mix that he has to deliver should have

1. not more than 75% gravel
2. minimum 60% and
3. Not more than 5% silt.

The pit 1 material consists of 70% gravel and 30% sand. The pit 2 material consists of 30% gravel, 60% sand and 10% silt.

* Formulate a minimum cost model.
* Determine the proportion of sand gravel and silt in the optimum solution.

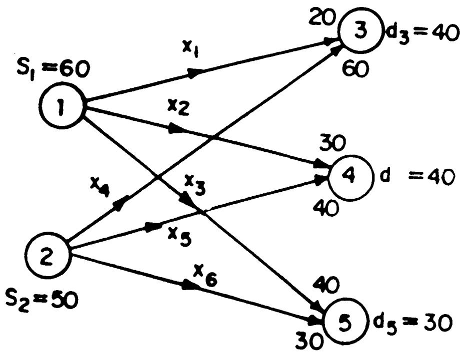
Solution:

(a) **Formulation:** We define the control variables to be

x_1 = amount of material taken from pit 1

x_2 = amount of material taken from pit 2

The cost function is



In case supply is less than the demand, inequality sign be used.

The network analysis can be used to many other types of problems also like to assign the equipments at different sites, to find out the maximum capacity of a water distribution system etc.

Example: From the grain size distribution it has been found out that the roadway aggregate should have the following specifications :

Aggregate specification

		Gravel	Sand	Silt
Percent volume	Lower limit	10	70	0
	Upper limit	20	90	20

Three sources of material are identified for the road construction from Pilani to Chirawa (14km)

Sources	% gravel	% sand	%silt	cost/m^3
Pilani	20	70	10	500
Chirawa	30	65	5	600
Rajgarh	—	100	—	800

The soil on which the construction has to be done contains 5% gravel, 25% sand and 70% silt supply 25000m^3 of aggregate

(a) Formulate a minimum cost aggregate mix model.

(b) Find out the minimum cost.

(c) Determine the highest price you charge from Pilani.

y_1 = amount of Pilani gravel (1000m^3)

y_2 = amount of Chirawa gravel (")

$$A = CM^{0.6} \qquad \text{(approximation)}$$

in which C is a constant. Furthermore, although approximation represents a smooth curve it only has very small curvature and the form

$$A = a + bM \qquad (3)$$

in which a and b are suitably chosen constants, approximates equations 2,

very closely throughout the range of available sections. Thus if equation 3 is substituted into the volume function (1) the following is obtained

$$V = 8(a + bM_1) + 2 \times 4\,(a + bM_2)$$

$$V = 16a + b(8M_1 + 8M_2) \qquad (4)$$

The element 16a in equation 4 is a constant as is the factor b. The only variable measure in equation 4 is the factor $(6M_1 + 6M_2)$. This therefore provides a measure of the cost of the frame which can be minimized. The entire optimum design process can then be expressed formally as

Minimize V $8M_1 + 8M_2$ over variables M_1, M_2 subject to non-violation of the inequalities

$$4M_1 > 120$$
$$M_1 + 3M_2 > 120$$
$$2M_1 + 2M_2 > 120$$
$$4M_2 > 120$$
$$4M_1 + 2M_2 > 240$$
$$2M_1 + 4M_2 > 240$$
$$M_1, M_2 > 0$$

Example : Allocating a Tower Crane

A contractor has work on 4 separate sites A, B, C and D and is considering hiring a large tower crane for 4 months to help in the construction work, because of the limited mobility of the crane it can only be moved from site to site at the end of each month. The crane is initial at the hiring yard, Y, and must be returned there after 4 months. The contractor has estimated the benefits to him in cash terms of using the crane on each of the 4 sites in each of the 4 months.

The lender has quoted a cost of 8000 to lend of the crane for 4 months, (the contractor to pay all moving costs). How should the contactor allocate the crane among his sites to maximize his total returns over the 4 months?

Knowing y, the moment of interia of the cross-section can be expressed as

$$I = \frac{1}{12} \quad (x_1x_3^3 + x_3x_4 + x_1x_6^3) + x_1x_2 \left(y - \frac{x_2}{2} \right)^2 + x_3x_4$$

$$(x_2 + \left(\frac{x_4}{2} - y \right)^2 + x_1x_5 \left(\quad x_2 + x_4 + \frac{x_5}{2} - y \right)^2$$

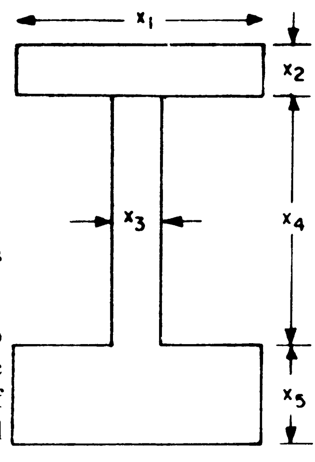
The maximum stress in the extreme fibres of the top and bottom flanges

$$(M_{max} Y)/I \text{ and } \quad \frac{M_{max}}{I} \quad (x_2 + x_4 + x_5 - 4)$$

The top flange is in compression and the bottom flange is in tension. Two stress constraints can be written:

$$\frac{M_{max} Y}{I} < f_c \text{ and } \frac{M_{max} Y}{I} < f_t$$

Where f_c and f_t are maximum permissible stress values determined from code. Stresses are not given as constants but depend on section properties. Such as slenderness ratio, l/r, (length/radius of gyration) and d/t (over-all depth/compression flange thickness).



The other constraints could be

* min thickness of plates
* Overall depth ($x_2 + x_4 + x_5$)
* Width of flange (x_1)
* Depth/width ratio $(x_2 + x_4 + x_5)/x_1$
* Provide vertical and being stiffness to prevent buckling.

The example was just to show how to design a mathematical model for a beam. The concept can be extended to other types of beam, columns, slabs and other structural members.

Considering next the shearing stresses, codes often prescribe a maximum average shearing stress, and a peak shearing stress, max, neither of which may be exceeded in the beam.

$$\frac{V}{(x_1x_2 + x_3x_4 + x_1x_5)} < \tau_{av}, \frac{fl}{2}$$