angular measurements of triangulation and traverse surveys, the astronomical results are used in calculating the size and shape of the Earth, and in determining the ellipsoidal surfaces to be used as datum surfaces for extended mapping systems. See Section 8.03.

1.03 Celestial Sphere. In field astronomy, use may be made of the Sun, Moon, and Planets, as well as the Stars. However, the stars are so numerous and make such excellent signals on which to direct the telescope, that the most precise results are obtainable from them. Moreover, stellar astronomy is easier to understand and simpler to compute, therefore consideration of other objects mentioned will be deferred till after the use of the stars has been discussed.

That stellar astronomy is comparatively simple is due mostly to the great distances of the stars. One effect of this is that the stars' own movements about in space are almost imperceptible, the familiar patterns of the constellations have remained the same for centuries, and we can identify each star by its position in the pattern. Another effect is that the directions of the stars as seen from the Earth are practically independent of the movements of the Earth or the position of an observer on the Earth: even the nearest star is so far away that the directions to it from two diametrically opposite points on the Earth will differ by less than 0·000 1 second. Because of these circumstances, it is permissible and convenient to regard the stars as point sources of light fixed on the inside of a colossal hollow sphere, the *celestial sphere*, having a radius effectively infinite, and the Earth at its centre.

Because there are so many stars, distributed all over the sky, the observer can choose those that are in the positions to give optimum accuracy of the calculated results.

1.04 Apparent Movement of Stars. Anyone who watches the stars on successive nights will become familiar with the pattern of the constellations and will also note that they pass steadily across the sky, night after night, each star apparently tracing out the same path each time. This phenomenon is of course simply an apparent movement due to the actual steady rotation of the Earth about its axis. An observer in middle northern latitudes, looking towards the northern parts of the sky, will see that the stars move round in circular concentric paths as shown in Fig. 1; the centre P of the tracks is of course a point where the Earth's axis of rotation meets the celestial sphere. For the present we can assume that the axis of rotation maintains a fixed direction in space (see Section 1.11); it is a particular diameter of the celestial sphere and it determines the positions of two *celestial poles*.

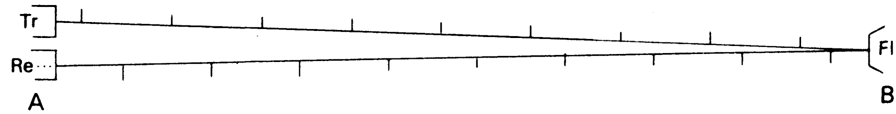

FIG. 49. IN PHASE.

just emerging from Tr therefore another pattern mark, the one 18 patterns ahead on the beam, is just entering Re. The situation a little later is represented in Fig. 49. A portion of a pattern has emerged from Tr and a portion of the 18th pattern has been absorbed into Re and is indicated by dots. A little consideration will show that at all times the two portions mentioned will be of equal length. Whatever the nature of the electromagnetic variations that make up a pattern, the variations entering Re will always be similar to those just coming from Tr. This situation is described by saying that the transmitted and received patterns are in phase, and this will occur when the length AB is a whole number of pattern lengths.

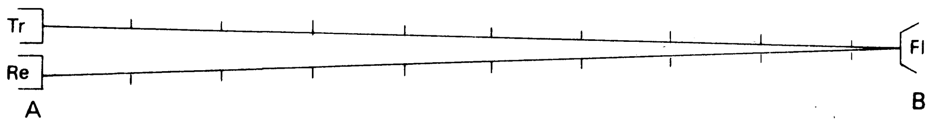

FIG. 50. MEASURING LENGTH.

Now consider Fig. 50. This shows the distance AB as $9\frac{1}{2}$ pattern lengths. It is obvious that when the distance is an integral number of half patterns, the return and outgoing signals will also be in phase. Thus the transmitted and received signals will be in phase whenever the distance AB is a multiple of $\frac{1}{2}p$. The effective unit of measurement in an instrument operating this principle is half the pattern length, which would be $2\frac{1}{2}$ metres in the numerical example used above.

By a suitable electronic device it is easy to detect when the signals are in phase, but of course the line to be measured will generally not be an exact multiple of $\frac{1}{2}p$, that is the transmitted and received signals will not generally be in phase.

Fig. 51 illustrates the general situation. The drawing shows the moment when a pattern mark is arriving at the receiver, and a later pattern mark has proceeded a distance s out of the transmitter,

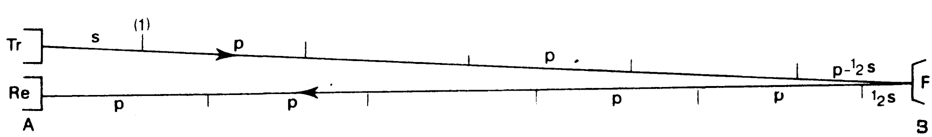

FIG. 51. PHASE COMPARISON.

electrical power supply, surveyors will have to be equipped with petrol-electric generator sets for battery charging.

4.13 Geometrical Corrections. In the Earth's atmosphere, the measuring beam will usually be curved upwards above the straight line, following the refracted path due to the decrease of density with increasing height. This path is shown dotted in Fig. 58, but the

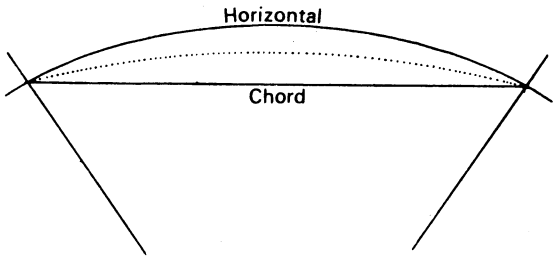

FIG. 58. RAY PATH.

length that is required in describing the geometry of a survey system and carrying out ordinary geodetic calculations of positions is the length of the curve having the Earth's radius.

If S is the length of an arc of radius R, then the length of the chord is

$$2R \sin\left(\frac{S}{2R}\right) = 2R\left(\frac{S}{2R} - \frac{S^3}{48R^3} \cdots\right) = S - \frac{S^3}{24R^2} \cdots .$$

The radius of the ray path for radio waves is not easily determined in the field; if we take a typical value of $4R$, then to get the geodetic length the measured length must be reduced to chord by subtracting $S^3/24(4R)^2$ and then increased by adding $S^3/24R^2$, where R is the radius of the Earth. The combined correction is thus about $+S^3/26R^2$.

For instance, if $S = 30$ km, $R = 6370$ km, the correction is $+26$ millimetres, which is less than one part in a million. Thus, the curvature correction is usually negligible, but it should be applied to lines over 50 km long.

Fig. 58 shows the line as at sea level; in practice the points will generally be at different heights, and the usual corrections for slope and for height will be necessary. For measurement of slopes the surveyor will need a theodolite and should measure the slope of each line in both directions.

4.14 Spreading the Observations. It is a feature of modern geodetic surveying instruments that their internal accuracy has reached a standard at which the principal sources of errors in observations theory: some well-known simple procedures are designed to achieve the requirements already mentioned by what may be called, for want of a better word, arbitrary methods. These methods have even been called 'semi-rigorous' whatever that means, but such descriptions may be misleading, since these methods, however arbitrary, do remove inconsistencies by unbiased procedures, and surely this is the prime object of all adjustment.

5.07 Arbitrary Methods. Perhaps the most obvious example of arbitrary adjustment comes also from traversing. It is quite common practice to adjust a traverse by first distributing the bearing misclosure evenly among the angles, then using these adjusted bearings to calculate coordinates. Misclosures generally appear, and adjustments are applied to the coordinates, increasing along the traverse until the adjustments at the closing point exactly cancel the misclosures. In the Bowditch method, for instance, the adjustment at each point is proportional to the accumulated length of the traverse lines up to that point. In any case, the lengths and bearings of the adjusted traverse, if required, must then be calculated from the adjusted coordinates of the stations. Some more examples of arbitrary methods of adjustment are described later.

5.08 The Idea of Weights. Reference has been made to 'unbiased' application of adjustments. This does not necessarily mean that misclosures must be equally divided. It may happen that some of the measurements used in a survey are considered to be more reliable than others, and it is easy to take account of this fact. Consider the triangle mentioned in Section 5.01, and suppose that one of the angles is known to be more reliable than the other two: the surveyor might reasonably decide that this angle should receive a smaller share of the total adjustment.

Perhaps he thinks that adjustments in the ratios 1:2:2 would be appropriate. A little arithmetical thought indicates adjustments of 0·8″, 1·6″, and 1·6″. This process is called weighting. See Section 5.17.

Again, in a traverse it might reasonably be thought that angles measured between short lines will be less reliable than those measured where the lines are longer. Suppose that there are 12 measured angles and 5 of these are thought to be less reliable than the rest; the bearing misclosure could be divided by 17, and 2/17 of the misclosure applied to the 5 weaker angles, 1/17 to each of the others.

In levelling, it is obvious that the reliability of a difference of level will decrease as the length of the line increases: this can be taken into account by a system of weighting. See Section 5.30.