in Fig. 6, we can again use the general solution, eq. (*b*), of the preceding article. Since the deflection and the bending moment approach zero as the distance x from the loaded end increases, we must take $A = B = 0$ in that solution. We obtain

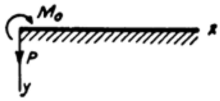

FIG. 6.

$$y = e^{-\beta x}(C \cos \beta x + D \sin \beta x). \qquad (a)$$

For determining the constants of integration C and D we have the conditions at the origin, i.e., under the load P:

$$EI_z \left(\frac{d^2y}{dx^2}\right)_{x=0} = -M_0,$$

$$EI_z \left(\frac{d^3y}{dx^3}\right)_{x=0} = -V = P.$$

Substituting from eq. (*a*) into these equations, we obtain two linear equations in C and D, from which

$$C = \frac{1}{2\beta^3 EI_z}(P - \beta M_0); \qquad D = \frac{M_0}{2\beta^2 EI_z}.$$

Substituting into eq. (*a*), we obtain

$$y = \frac{e^{-\beta x}}{2\beta^3 EI_z}[P \cos \beta x - \beta M_0(\cos \beta x - \sin \beta x)] \qquad (11)$$

or, using notations (6),

$$y = \frac{2\beta}{k}\{P\theta(\beta x) - \beta M_0[\theta(\beta x) - \zeta(\beta x)]\}.$$

To get the deflection under the load we must substitute $x = 0$ into eq. (11). Then

$$\delta = (y)_{x=0} = \frac{1}{2\beta^3 EI_z}(P - \beta M_0). \qquad (11')$$

The expression for the slope is obtained by differentiating eq. (11). At the end ($x = 0$) this becomes

$$\left(\frac{dy}{dx}\right)_{x=0} = -\frac{1}{2\beta^2 EI_z}(P - 2\beta M_0). \qquad (12)$$

By using eqs. (11′) and (12) in conjunction with the principle of superposition, more complicated problems can be solved. Take as an example a uniformly loaded long beam on an elastic foundation, Substituting this value into eq. (k), we find the reaction at the middle support of the vertical beam, which intersects the beam AB at its mid-point. It is interesting to note that this reaction may become negative, which indicates that the horizontal beam actually supports the vertical beams only if it is sufficiently rigid; otherwise it may increase the bending of some of the vertical beams.

Problems

1. Find a general expression for the deflection curve for the beam illustrated in Fig. 12.

Answer.

$$y = \frac{2P\beta}{k} \frac{\cosh \beta x \cos \beta(l - x) + \cosh \beta(l - x) \cos \beta x}{\sinh \beta l + \sin \beta l}.$$

2. Find the deflections at the ends and the bending moment at the middle of the beam bent by two equal and opposite couples M_0, Fig. 18.

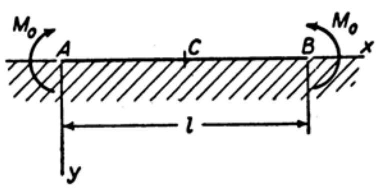

FIG. 18.

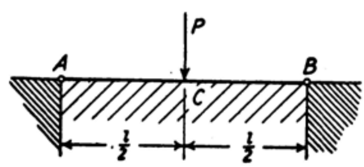

FIG. 19.

Answer.

$$y_a = y_b = -\frac{2M_0\beta^2}{k} \frac{\sinh \beta l - \sin \beta l}{\sinh \beta l + \sin \beta l},$$

$$M_c = 2M_0 \frac{\sinh \dfrac{\beta l}{2} \cos \dfrac{\beta l}{2} + \cosh \dfrac{\beta l}{2} \sin \dfrac{\beta l}{2}}{\sinh \beta l + \sin \beta l}.$$

3. Find the deflection and the bending moment at the middle of the beam with hinged ends, Fig. 19. The load P is applied at the middle of the beam.

Answer.

$$y_c = \frac{P\beta}{2k} \frac{\sinh \beta l - \sin \beta l}{\cosh \beta l + \cos \beta l},$$

$$M_c = \frac{P}{4\beta} \frac{\sinh \beta l + \sin \beta l}{\cosh \beta l + \cos \beta l}.$$

left end of the strut is

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{ql}{2S}\left(\frac{\tan\frac{pl}{2}}{\frac{pl}{2}} - 1\right) = \frac{ql^3}{24EI}\frac{\tan u - u}{\frac{1}{3}u^3}. \qquad (32)$$

The maximum bending moment is at the middle where

$$M_{\max} = -EI\left(\frac{d^2y}{dx^2}\right)_{x=l/2}$$

$$= EI\frac{q\left(1 - \cos\frac{pl}{2}\right)}{S\cos\frac{pl}{2}} = \frac{ql^2}{8}\cdot\frac{2(1-\cos u)}{u^2\cos u}. \qquad (33)$$

By using the solution for the case of a couple together with the solutions for lateral loads and applying the method of superposition, various statically indeterminate cases of bending of struts can be readily solved. Taking as an example the case of a uniformly loaded strut built in at one end, Fig. 27,

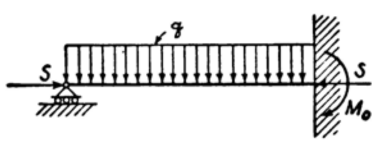

FIG. 27.

we find the bending moment M_0 at the built-in end from the condition that this end does not rotate during bending. By using eqs. (28) and (32) this condition is found to be

$$-\frac{ql^3}{24EI}\frac{\tan u - u}{\frac{1}{3}u^3} + \frac{M_0 l}{3EI}\cdot\left(\frac{3}{2u\tan 2u} - \frac{3}{(2u)^2}\right) = 0$$

from which

$$M_0 = -\frac{ql^2}{8}\cdot\frac{4\tan 2u\,(\tan u - u)}{u(\tan 2u - 2u)}. \qquad (34)$$

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