Subsequently i_t decays exponentially as before. The current following circuit switching is therefore $i = i_s + i_t$, giving

$$i = (v_m/Z)\,[\sin(\omega t - \theta) - \sin(\omega t_0 - \theta)\,.\,\exp(-R/L)(t - t_0)].$$

In machine circuits R is often much smaller than X so that $Z \simeq X$ and $\theta \simeq 90°$. For these conditions, approximately,

$$i \simeq (v_m/X)[-\cos\omega t + \cos\omega t_0\,.\,\exp(-R/L)(t - t_0)].$$

If the voltage is switched on at a zero ($t_0 = 0, \pi/\omega, 2\pi/\omega\ldots$), the transient term has its greatest value v_m/X, and the resultant current

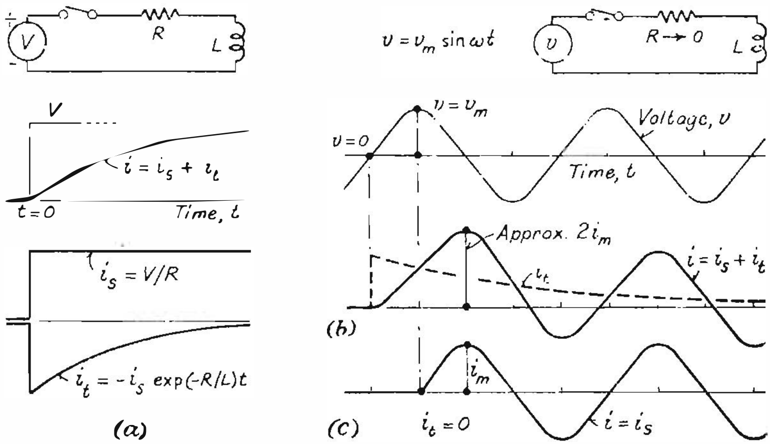

FIG. 1. TRANSIENT PHENOMENA IN RL CIRCUIT

starts from zero with complete asymmetry, Fig. 1 (*b*). In contrast, if the switch is closed on peak voltage, the resultant current attains steady state instantaneously without any transient, Fig. 1 (*c*). The current in (*b*) has an amplitude nearly double that in (*c*), an example of the *doubling effect*. Intermediate switching instants give partial asymmetry, with smaller transient components.

TRANSIENT AND STEADY STATES. It is usual to develop circuit theory on a steady-state basis, using complex algebraic treatment with complexor or "vector" diagrams. The consideration of transient conditions demands a return to more basic concepts.

STEADY-STATE CONVENTIONS. A complexor* voltage $\boldsymbol{V}$, of magnitude V and drawn at an arbitrary angle α to a horizontal datum, can be variously described as

$$\boldsymbol{V} = V\underline{/\alpha} = V_1 + jV_2 = V(\cos\alpha + j\sin\alpha) = V\,.\,\exp(j\alpha).$$

* Voltages and currents are not vectors in the true physical sense, and are here called "complexors." However, in deference to common usage the term "vector" is also occasionally used in the text.

If another circuit (the *secondary*) be in the vicinity of the first (the *primary*), it will link some of the magnetic flux produced by the primary (Fig. 7). With an alternating primary current (and therefore flux) the changing linkages will produce in the secondary an e.m.f.

$$e_{2M} = -\frac{dN_{2M}}{dt} \text{ volts}$$

where N_{2M} represents the linkages in the T_2 turns of the secondary winding with that part Φ_{1M} of the flux Φ_1 produced by the primary that links the secondary. If the secondary coil is suitably shaped and favourably placed relatively to the primary, $N_{2M} \simeq T_2\Phi_{1M}$: in general N_{2M} will differ from this simple product as it is not possible to secure that *all* the flux Φ_{1M} links *all* the turns T_2 completely.

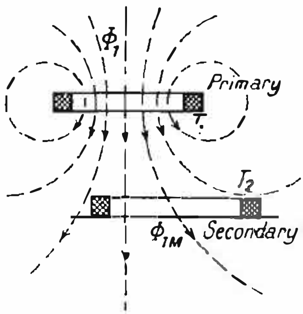

FIG. 7. MUTUAL INDUCTION

The e.m.f. e_{2M} is said to be produced by reason of the *mutual induction* of the primary and secondary circuits. A similar effect will naturally take place if the respective roles of the two circuits are interchanged, and it is shown in textbooks of electrical technology that the mutual inductance is the same irrespective of which circuit is primary and which secondary, in any given case. The coefficient of mutual inductance L_{12} in henrys may be defined as the e.m.f. in volts induced in one circuit when the current in the other is changed at the rate of 1 A. per sec.; or the energy in the common magnetic field in joules when each circuit carries 1 A.

The mutually induced e.m.f. in the secondary circuit will, if the circuit be closed through a load, circulate current in the load and dissipate energy therein. This energy can come only from the primary, to which the whole operation is due. Thus energy is being transferred from primary to secondary by means of the mutual magnetic field. This is important, and is the principle underlying the transformer effect. The process briefly is: the primary produces a pulsating magnetic field in which energy is stored and restored periodically. The e.m.f. e_{2M} and the current i_2 associated with it in the secondary circuit abstract energy from the common field and pass it on to the secondary load. If there is no secondary load the magnetic field energy passes into and out of the primary circuit as a continual pulsation of energy from electrical to and from magnetic form

The more closely the primary and secondary circuits are mutually linked, the more direct becomes the exchange of energy between them. If the two circuits link a common iron core, Fig. 8, the effects are—

Leakage between primary and secondary could be eliminated if the windings could be made to occupy the same space. This, of course, is physically impossible, but an approximation to it is achieved if the coils of primary and secondary are sectionalized and interleaved: such an arrangement leads to a marked reduction of the leakage reactance. If, on the other hand, the primary and secondary are kept separate and widely spaced, there will be much more room for leakage flux and the leakage reactance will be greater. It is thus possible to control the reactance within limits. The calculation of reactance is detailed in Chapter VII, §7.

THE EQUIVALENT CIRCUIT. The transformer shown diagrammatically in Fig. 15 (*a*) can be resolved into an equivalent circuit (*b*) in which the resistance and leakage reactance of primary and

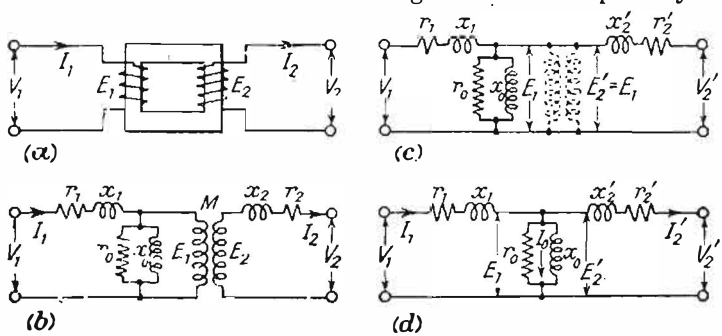

FIG. 15. EQUIVALENT CIRCUIT

secondary respectively are represented by the "lumped" r_1, x_1, r_2 and x_2, as if these were external to a transformer of which the windings were without resistance and leakage. Similarly a shunt circuit r_0 and x_0 can be introduced such that $E_1/r_0 = I_{0a}$ and $E_1/x_0 = I_{0r}$, the two quadrature components of the magnetizing current. The windings of the transformer are now "ideal," and represent the seat of the induced e.m.f.'s E_1 and E_2, which are related by the expression $E_1/E_2 = T_1/T_2$, the turn-ratio.

Suppose $T_1 = T_2$, then $E_1 = E_2$, and the two sides of the transformer may be joined in parallel (*c*), and the energy transmitted from primary to secondary without a transformer at all (*d*). The circuit, Fig. 15 (*d*), represents exactly the electrical characteristics of a transformer with unity turn-ratio: that is, the resistance and reactance voltages, no-load current, core and I^2R losses, are reproduced and give the same characteristics as the transformer.

An equivalent circuit is useful for calculations of regulation, parallel operation, etc. Since in the majority of cases the turn-ratio is not unity, it is necessary to imagine the actual secondary winding of T_2 turns replaced by an *equivalent* winding of T_1 turns, for which the I^2R loss and the *per-unit* or *percentage* reactance

ϕ, the regulation given by eq. (19) is $(\overline{BC} + \overline{CD})/\overline{OA} = \overline{BD}/\overline{OA}$. The true regulation is $\overline{BA}/\overline{OA}$, so that it is necessary to make the small addition $\overline{DA}/\overline{OA}$. This term is

$$\frac{\overline{DA}}{\overline{OA}} \simeq \frac{\overline{FD}^2}{2\overline{OA}^2} = \frac{(\overline{FH} - \overline{DH})^2}{2\overline{OA}^2} = \frac{(I_2X_2 \cos\phi - I_2R_2 \sin\phi)^2}{2V_1'^2},$$

whence eq. (20). In Fig. 21, (*a*) is drawn in primary and (*b*) in secondary terms: both naturally lead to the same result.

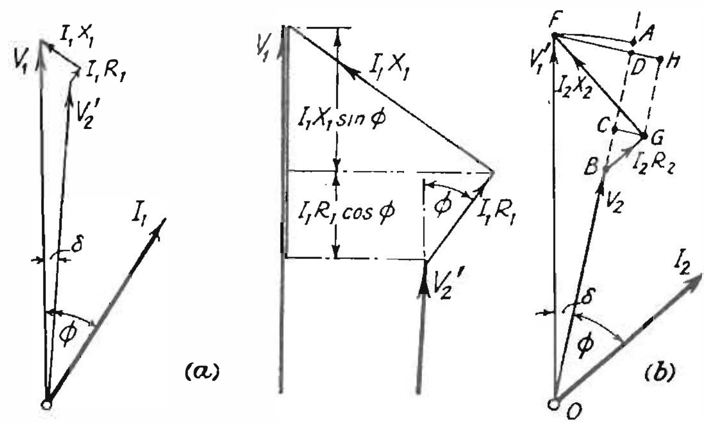

FIG. 21. CALCULATION OF REGULATION

Eq. (19) indicates that the regulation on full load varies with the power factor at the secondary terminals. It will be a maximum when $\phi = \text{arc tan } (\varepsilon_x/\varepsilon_r)$, as can be seen from Fig. 21 (*a*), where the greatest difference between V_1 and V_2' will occur when the angle ϕ coincides with the internal angle arc tan $(X_1/R_1) = \text{arc tan } (\varepsilon_x/\varepsilon_r)$ of the total impedance. The regulation will be zero when $\varepsilon_r \cos\phi + \varepsilon_x \sin\phi = 0$, i.e. when $\tan\phi = -(\varepsilon_r/\varepsilon_x)$, giving $\phi = -\text{arc tan } (\varepsilon_r/\varepsilon_x)$ corresponding to a negative (leading) angle. At leading power factors below this the regulation will be negative, i.e. the secondary terminal voltage will *rise* between no load and full load. The full-load regulation at various power-factors is shown for a typical case in Fig. 22; while Fig. 20 provides a vectorial explanation, the impedance drop $I_1(z_1 + z_2') = I_1Z_1$ being exaggerated for clarity.

The numerical values ε_r and ε_x are readily calculable from the short-circuit test, as described in Chapter VI, §7.

Regulation is a numeric, not a complex quantity, so that the

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INSETS