The natural frequency for this system is then given by

$$\omega = \sqrt{k_e/m}$$

$$\omega = \sqrt{9.07 \times 386/50.7}$$

$$\omega = 8.31 \text{ rad/sec}$$

or

$$f = 1.32 \text{ cps.} \qquad \text{(Ans.)}$$

1.9 AMPLITUDE OF MOTION

Let us now examine in more detail eq. (1.20), the solution describing the free vibratory motion of the undamped oscillator. A simple trigonometric transformation may show us that we can rewrite this equation in the equivalent forms, namely

$$y = C \sin(\omega t + \alpha) \tag{1.23}$$

or

$$y = C \cos(\omega t - \beta) \tag{1.24}$$

where

$$C = \sqrt{y_0^2 + (v_0/\omega)^2}, \tag{1.25}$$

$$\tan \alpha = \frac{y_0}{v_0/\omega}, \tag{1.26}$$

and

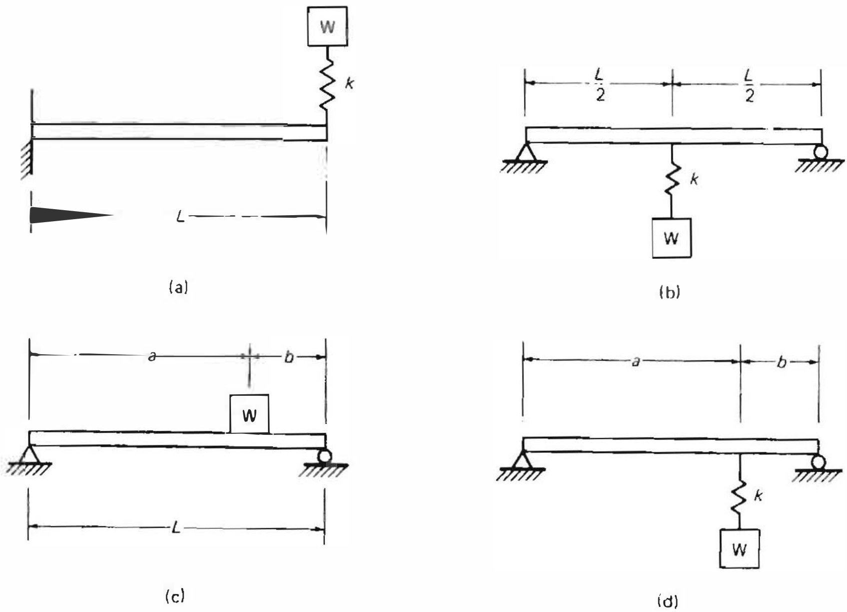
$$\tan \beta = \frac{v_0/\omega}{y}. \tag{1.27}$$

The simplest way to obtain eq. (1.23) or eq. (1.24) is to multiply and divide eq. (1.20) by the factor C defined in eq. (1.25) and to define α (or β) by eq. (1.26) [or eq. (1.27)]. Thus

$$y = C\left(\frac{y_0}{C} \cos \omega t + \frac{v_0/\omega}{C} \sin \omega t\right). \tag{1.28}$$

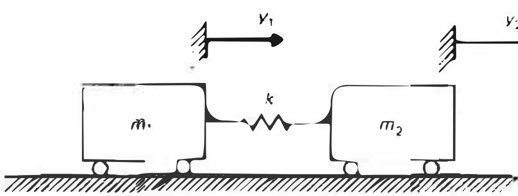
With the assistance of Fig. 1.9, we recognize that

$$\sin \alpha = \frac{y_0}{C} \tag{1.29}$$



P1.10

1.11 A system (see Fig. P1.11) is modeled by two freely vibrating masses m_1 and m_2 interconnected by a spring having a constant k. Determine for this system the differential equation of motion for the relative displacement $u = y_2 - y_1$ between the two masses. Also determine the corresponding natural frequency of the system.



P1.11

The value of the damping coefficient for real structures is much less than the critical damping coefficient and usually ranges between 2 to 20% of the critical damping value. Substituting for the maximum value $\xi = 0.20$ into eq. (2.17),

$$\omega_D = 0.98\,\omega. \tag{2.25}$$

It can be seen that the frequency of vibration for a system with as much as a 20% damping ratio is essentially equal to the undamped natural frequency. Thus, in practice, the natural frequency for a damped system may be taken to be equal to the undamped natural frequency.

2.6 LOGARITHMIC DECREMENT

A practical method for determining experimentally the damping coefficient of a system is to initiate free vibration, obtain a record of the oscillatory motion, such as the one shown in Fig. 2.4, and measure the rate of decay of the amplitude of motion. The decay may be conveniently expressed by the *logarithmic decrement* δ which is defined as the natural logarithm of the ratio of any two successive peak amplitudes, y_1 and y_2 in free vibration, that is,

$$\delta = \ln \frac{y_1}{y_2}. \tag{2.26}$$

The evaluation of damping from the logarithmic decrement follows. Consider the damped vibration motion represented graphically in Fig. 2.4 and given analytically by eq. (2.21) as

$$y(t) = C e^{-\xi\omega t} \cos(\omega_D t - \alpha).$$

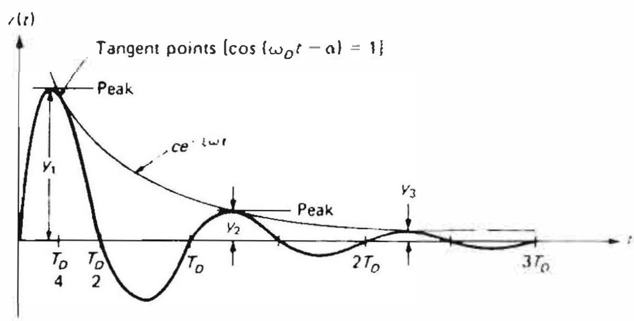


Fig. 2.4 Curve showing peak displacements and displacements at the points of tangency.

decrease 5% on each consecutive cycle of motion. Determine the damping coefficient c of the system. $k = 200$ lb/in and $m = 10$ lb · sec^2/in.

2.4 It is observed experimentally that the amplitude of free vibration of a certain structure, modeled as a single degree-of-freedom system, decreases from 1 to 0.4 in 10 cycles. What is the percentage of critical damping?

2.5 Show that the displacement for critical and overcritical damped systems with initial displacement y_0 and velocity v_0 may be written as

$$y = e^{-\omega t}\,[y_0(1+\omega t) + v_0 t] \qquad \text{for } \xi = 1$$

$$y = e^{-\xi\omega t}\left[y_0 \cosh \omega_D' t + \frac{v_0 + y_0\xi\omega}{\omega_D'} \sinh \omega_D' t\right] \qquad \text{for } \xi > 1$$

where $\omega_D' = \omega\sqrt{\xi^2 - 1}$.

2.6 A structure is modeled as a damped oscillator with spring constant $k = 30$ Kips/in and undamped natural frequency $\omega = 25$ rad/sec. Experimentally it was found that a force 1 Kip produced a relative velocity of 1.0 in/sec in the damping element. Find: (a) the damping ratio ξ, (b) the damped period T_D, (c) the logarithmic decrement δ, and (d) the ratio between two consecutive amplitudes.

2.7 In Fig. 2.4 it is indicated that the tangent points to the displacement curve correspond to $\cos(\omega_D t - \alpha) = 1$. Therefore the difference in $\omega_D t$ between any two consecutive tangent points is 2π. Show that the difference in $\omega_D t$ between any two consecutive peaks of the curve is also 2π.

2.8 Show that for an underdamped system in free vibration the logarithmic decrement may be written as

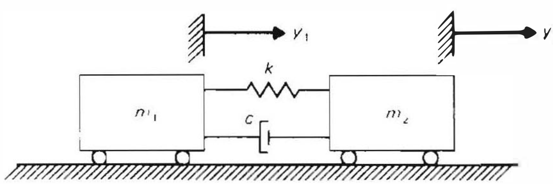
$$\delta = \frac{1}{k} \ln \frac{y_i}{y_{i+k}}$$

where k is the number of cycles separating two measured peak amplitudes y_i and y_{i+k}.

2.9 A single degree-of-freedom system consists of a mass with a weight of 386 lb and a spring of stiffness $k = 3000$ lb/in. By testing the system it was found that a force of 100 lb produces a relative velocity 12 in./sec. Find, (a) the damping ratio ξ, (b) the damped frequency of vibration f_D, (c) logarithmic decrement δ, and (d) the ratio of two consecutive amplitudes.

2.10 Solve Problem 2.9 when the damping coefficient is $c = 2$ lb sec/in.

2.11 A system is modeled by two freely vibrating masses m_1 and m_2 interconnected by a spring and a damper element as shown in Fig. P2.11. Deter-



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