1-12 Determine the Fourier series for the sawtooth curve shown in Fig. P1-12. Express the result of Prob. 1-12 in the exponential form of Eq. (1.2-4).

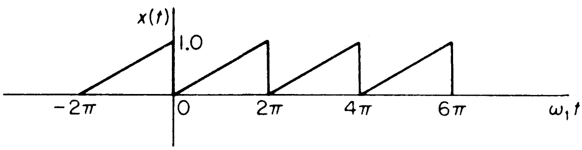


Figure P1-12.

1-13 Determine the rms value of a wave consisting of the positive portions of a sine wave.

1-14 Determine the mean square value of the sawtooth wave of Prob. 1-12. Do this two ways, from the squared curve and from the Fourier series.

1-15 Plot the frequency spectrum for the triangular wave of Prob. 1-11.

1-16 Determine the Fourier series of a series of rectangular pulses shown in Fig. P1-16. Plot c_n and ϕ_n versus n when $k = \frac{2}{3}$.

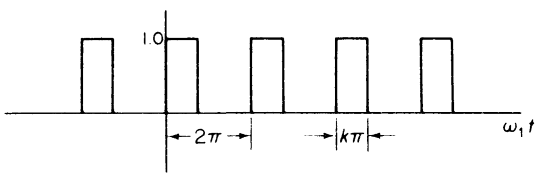
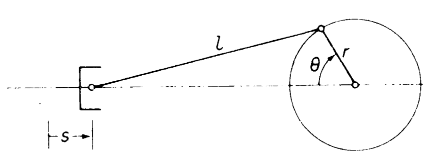


Figure P1-16.

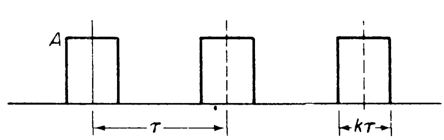
1-17 Write the equation for the displacement s of the piston in the crank-piston mechanism shown in Fig. P1-17, and determine the harmonic components and their relative magnitudes. If $r/l = \frac{1}{3}$, what is the ratio of the second harmonic compared to the first?

Figure P1-17.



1-18 Determine the mean square of the rectangular pulse shown in Fig. P1-18 for $k = 0.10$. If the amplitude is A, what would an rms voltmeter read?

Figure P1-18.



Since $\delta\theta$ is arbitrary, the quantity within the brackets must be zero. Thus the equation of motion becomes

$$\ddot{\theta} + \left(1 + \frac{2m_2}{m_1}\right)\frac{g}{l}\theta = 0$$

where $\sin\theta \cong \theta$ has been substituted. The natural frequency from the preceding equation is

$$\omega_n = \sqrt{\left(1 + \frac{2m_2}{m_1}\right)\frac{g}{l}}$$

2.6 VISCOUSLY DAMPED FREE VIBRATION

Viscous damping force is expressed by the equation

$$F_d = c\dot{x} \tag{2.6-1}$$

where c is a constant of proportionality. Symbolically it is designated by a dashpot, as shown in Fig. 2.6-1. From the free-body diagram the equation of motion is seen to be

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{2.6-2}$$

The solution of the above equation has two parts. If $F(t) = 0$, we have the homogeneous differential equation whose solution corresponds physically to that of *free-damped vibration*. With $F(t) \neq 0$, we obtain the particular solution that is due to the excitation irrespective of the homogeneous solution. We will first examine the homogeneous equation that will give us some understanding of the role of damping.

With the homogeneous equation

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{2.6-3}$$

the traditional approach is to assume a solution of the form

$$x = e^{st} \tag{2.6-4}$$

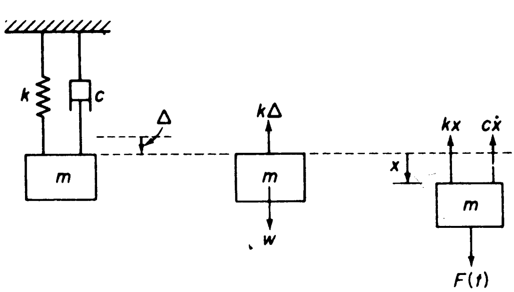
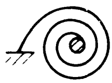
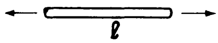
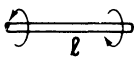
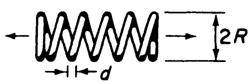
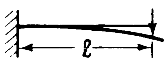
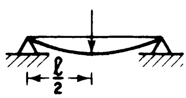
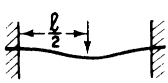
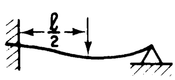
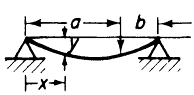
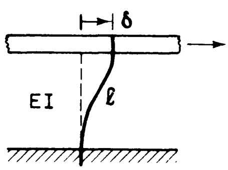
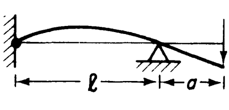
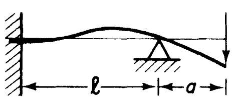


Figure 2.6-1.

	$k = \dfrac{EI}{l}$,	I = moment of inertia of cross-sectional area l = total length
	$k = \dfrac{EA}{l}$	A = cross-sectional area
	$k = \dfrac{GJ}{l}$	J = torsion constant of cross section
	$k = \dfrac{Gd^4}{64nR^3}$	n = number of turns
	$k = \dfrac{3EI}{l^3}$	k at position of load
	$k = \dfrac{48EI}{l^3}$	
	$k = \dfrac{192EI}{l^3}$	
	$k = \dfrac{768EI}{7l^3}$	
	$k = \dfrac{3EIl}{a^2b^2}$	$y_x = \dfrac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \dfrac{12EI}{l^3}$	
	$k = \dfrac{3EI}{(l + a)a^2}$	
	$k = \dfrac{24EI}{a^2(3l + 8a)}$	

3

HARMONICALLY EXCITED VIBRATION

When a system is subjected to harmonic excitation, it is forced to vibrate at the same frequency as that of the excitation. Common sources of harmonic excitation are unbalance in rotating machines, forces produced by reciprocating machines, or the motion of the machine itself. These excitations may be undesirable for equipment whose operation may be disturbed or for the safety of the structure if large vibration amplitudes develop. Resonance is to be avoided in most cases, and to prevent large amplitudes from developing, dampers and absorbers are often used. Discussion of their behavior is of importance for their intelligent use. Finally, the theory of vibration measuring instruments is presented as a tool for vibration analysis.

3.1 FORCED HARMONIC VIBRATION

Harmonic excitation is often encountered in engineering systems. It is commonly produced by the unbalance in rotating machinery. Although pure harmonic excitation is less likely to occur than periodic or other types of excitation, understanding the behavior of a system undergoing harmonic excitation is essential in order to comprehend how the system will respond to more general types of excitation. Harmonic excitation may be in the form of a force or displacement of some point in the system.

We will first consider a single-DOF system with viscous damping, excited by a harmonic force $F_0 \sin \omega t$ as shown in Fig. 3.1-1. Its differential