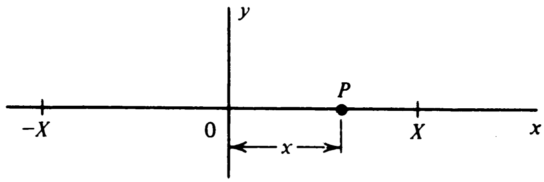


FIG. 1-5. *Simple harmonic motion:* $x(t) = X \cos \omega t$.

that is,

$$\text{Period } \tau = \frac{2\pi}{\omega} \text{ s/cycle} \tag{1-2}$$

$$\text{Frequency } f = \frac{1}{\tau} = \frac{\omega}{2\pi} \text{ cycle/s, or Hz*} \tag{1-3}$$

ω is called the circular frequency measured in rad/s.

If $x(t)$ represents the displacement of a mass in a vibratory system, the velocity and the acceleration are the first and the second time derivatives of the displacement,† that is,

$$\text{Displacement } x = X \cos \omega t \tag{1-4}$$

$$\text{Velocity } \dot{x} = -\omega X \sin \omega t = \omega X \cos(\omega t + 90°) \tag{1-5}$$

$$\text{Acceleration } \ddot{x} = -\omega^2 X \cos \omega t = \omega^2 X \cos(\omega t + 180°) \tag{1-6}$$

These equations indicate that the velocity and acceleration of a harmonic displacement are also harmonic of the same frequency. Each differentiation changes the amplitude of the motion by a factor of ω and the *phase angle* of the circular function by 90°. The phase angle of the velocity is 90° leading the displacement and the acceleration is 180° leading the displacement.

Simple harmonic motion can be defined by combining Eqs. (1-4) and (1-6).

$$\ddot{x} = -\omega^2 x \tag{1-7}$$

where ω^2 is a constant. When the acceleration of a particle with rectilinear motion is always proportional to its displacement from a fixed point on the path and is directed towards the fixed point, the particle is said to have simple harmonic motion. It can be shown that the solution of Eq. (1-7) has the form of a sine and a cosine function with circular frequency equal to ω.

* In 1965, the Institute of Electrical and Electronics Engineers, Inc. (IEEE) adopted new standards for symbols and abbreviation (IEEE Standard No. 260). The unit hertz (Hz) replaces cycles/sec (cps) for frequency. Hz is now commonly used in vibration studies.

† The symbols $\dot{x}$ and $\ddot{x}$ represent the first and second time derivatives of the function $x(t)$, respectively. This notation is used throughout the text unless ambiguity may arise.

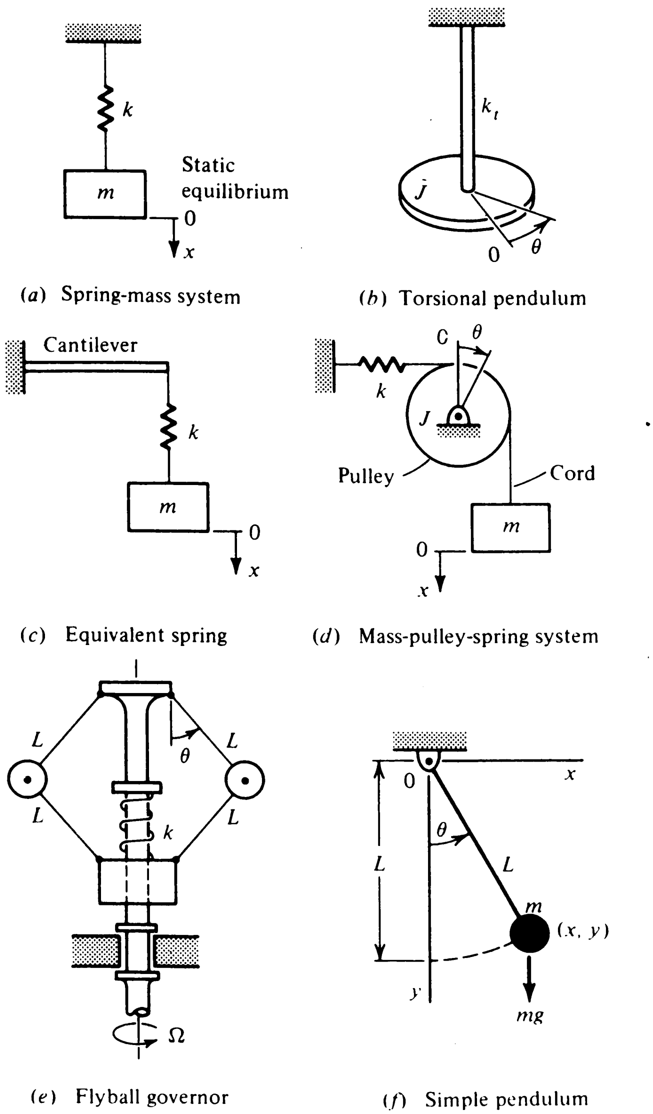


Fig. 2-1. *Examples of systems with one degree of freedom.*

time domain analysis, since the motion of the mass is a time function, such as the solution of a differential equation with time as the independent variable. The frequency response method assumes that both the excitation and the system response are sinusoidal and of the same frequency. Hence it is a *frequency domain analysis*. Note that time response is intuitive but it is more convenient to describe a system in the frequency domain.

Assuming $\zeta \le 0.1$ and solving for r, we get

$$r_{1,2} \simeq 1 \pm \zeta \tag{2-59}$$

$$\text{Bandwidth} = r_2 - r_1 = \frac{\omega_2}{\omega_n} - \frac{\omega_1}{\omega_n} = 2\zeta \tag{2-60}$$

A factor Q is also used to define the bandwidth and damping.

$$Q = \frac{1}{\text{bandwidth}} = \frac{1}{2\zeta} \tag{2-61}$$

Q is used to measure the *quality* of a resonance circuit in electrical engineering. It is also useful for determining the equivalent viscous damping in a mechanical system.

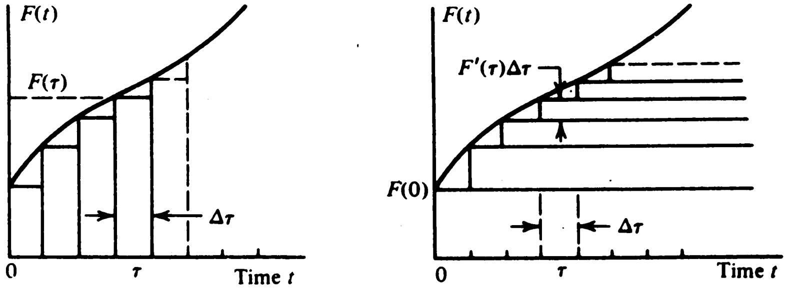
2-7 TRANSIENT VIBRATION

We shall show that the transient vibration due to an arbitrary excitation $F(t)$ can be obtained by means of superposition. Although the method is not convenient for hand calculations, it can be implemented readily using computers.

The equation of motion of the model in Fig. 2-6 for systems with one degree of freedom and an excitation $F(t)$ is

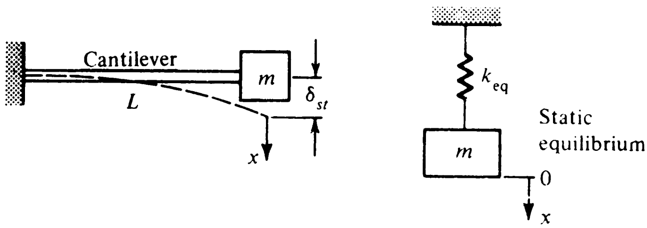
$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{2-62}$$

One method to solve the equation is to approximate $F(t)$ by a sequence of pulses as shown in Fig. 2-20(a). If the system response to a typical pulse input is known, the response to $F(t)$ can be obtained by superposition. In other words, the system response to $F(t)$ is the sum of the responses due to each of the pulses in the sequence.



(a) Sequence of pulses (b) Sequence of steps

FIG. 2-20. *F(t) approximated by pulses and steps.*



(*a*) Vibratory system (*b*) Equivalent system

FIG. 3-3. *Equivalent spring.*

Similarly, equivalent springs can be calculated. Let k_{t2} be the torsional stiffness of shaft 2. It can be shown by equating potential energies that the equivalent stiffness of shaft 2 referring to shaft 1 is n^2k_{t2}. The equivalent spring in a system can assume various forms. We shall illustrate the equivalent spring with the examples to follow.

Example 3. Equivalent Spring

Figure 3-3 shows that the static deflection δ_{st} of a cantilever beam is due to the mass m attached to its free end. Find the natural frequency of the system.

Solution:

The equivalent system is as shown in Fig. 3-3(b) if (1) the cantilever is of negligible mass and (2) m is small in size compared with L. The static deflection δ_{st} due to the concentrated force mg at the free end of a beam of length L is

$$\delta_{st} = \frac{mgL^3}{3EI}$$

where EI is the flexural stiffness of the beam. The equivalent spring constant k_{eq} is defined as force per unit deflection.

$$k_{eq} = \frac{mg}{\delta_{st}} = \frac{3EI}{L^3}$$

From the equivalent system, the natural frequency is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{3EI}{mL^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}$$

Example 4. Springs in Series

Springs are said to be *in series* when the deformation of the equivalent spring k_{eq} is the sum of their deformations. Assume the cantilever in Fig. 3-4(a) is of negligible mass. Show that the cantilever and the spring k_2 are in series.