position where they disappear one by one. The pattern seems to collapse to the center.

In contrast, if d is increased, the order of central fringe increases. Fringes of higher orders emerge from the central point and the interference pattern appears expanding.

In general, suppose when the difference between the mirror arms is d, the central point is dark fringe of order m; when the gap d is reduced to d', let the central dark fringe be of order $(m + N)$, i.e., N fringes have collapsed at the centre. Hence, we have

$$2d = m\lambda$$

and
$$2d' = (m + N)\lambda$$

From these relations, we find

$$\lambda = \frac{2(d'-d)}{N} = \frac{2\Delta d}{N} \qquad \text{...(4.15)}$$

Thus by measuring the distance Δd by which the movable mirror is displaced and observing the number N of fringes that appear to collapse at the centre, we can determine the wavelength of light.

Example 4.13

When the movable mirror of a Michelson interferometer is displaced by 0.0589 mm, 200 fringes are observed to cross the field of view. Determine the wavelength of light used.

Solution

According to Eq. 4.15, the wavelength is given by

$$\lambda = \frac{2\Delta d}{N}$$

where N is the number of fringes which pass the cross-wires of the viewing telescope when the mirror is displaced by Δd. Given that $N = 200$, and $\Delta d = 589 \times 10^{-7}$ m, we get

$$\lambda = \frac{2 \times 589 \times 10^{-7}}{200} = 5890 \text{ Å}$$

Example 4.14

When a thin sheet of a transparent material of refractive index 1.43 is inserted in one of the arms of Michelson interferometer, 26 fringes are found to shift across the field of view. It wavelength of light used is 5890 Å, calculate the thickness of the sheet.

Solution

Suppose the thickness of the sheet is t. When the sheet is inserted monochromaticity of light. Recall that more monochromatic a beam, greater is its temporal coherence. We have earlier seen that in case of thin films, if the path difference between two interfering waves exceed the longitudinal coherence length then no observable fringe pattern is obtained. Similarly, in case of Michelson interferometer, the coherence length or spectral width $\Delta\lambda$ of the source determines the limit for the difference between mirror arms ($d = |l_2 - l_1|$) beyond which no observable interference pattern is formed. This limit on d is given by

$$2d \leq l_l = \frac{\lambda^2}{\Delta\lambda}$$

or

$$d_{max} = \frac{\lambda^2}{2\Delta\lambda} \qquad \ldots(4.19)$$

where l_l is coherence length of light. Note that above equation, Eq. 4.19 is same as Eq. 4.6 for thin films (with $n_f = 1$, $\cos\theta_r = 1$); both express the fact that if path difference exceeds coherence length then interfering waves become incoherent and hence visibility vanishes.

Eq. 4.19 was written on the basis of the concept of coherence length. We can derive this condition from the primary fact that if the source has a continuous spectral width $\Delta\lambda$ about λ, then the fringe patterns due to different wavelengths would overlap and destroy the interference pattern if path difference exceeds the value ($\lambda^2/\Delta\lambda$). Let us now show this result.

Suppose the incident light contains continuous wavelengths from λ to ($\lambda + \Delta\lambda$). For simplicity, we may assume that all the wavelengths are of equal intensity. Now, each component wavelength produces its own interference pattern. If $d = 0$, all the wavelengths satisfy condition of destructive interference and we do not see any light intensity on the screen (focal plane of the lens). As d is increased, sharp fringe pattern appears. On further increase of d, fringes corresponding to different wavelengths shift on the screen and we expect that the pattern is finally completely smeared out.

Consider that for a value of d, at the position of minimum intensity for λ, maximum intensity is produced by ($\lambda + \Delta\lambda/2$); that is suppose we have

$$2d = n\lambda$$

$$= \left(n - \frac{1}{2}\right)(\lambda + \Delta\lambda/2)$$

In this situation the fringe pattern of first-half-spectral-width would mutually destroy the pattern of second-half. Eliminating *n*, we find that pattern is destroyed when

$$2d\left(\frac{1}{\lambda}-\frac{1}{\lambda+\Delta\lambda/2}\right)=\frac{1}{2}$$

or $$d=\frac{\lambda^2}{2\Delta\lambda}$$

where we neglect $\lambda\Delta\lambda/2$ in comparison to λ^2. The above is the same condition which we obtained earlier by arguments based on temporal coherence.

4.6 MULTIPLE BEAM INTERFERENCE IN PARALLEL FILM

So far we have discussed the phenomenon of interference between only two beams which were produced from a single beam either by division of wavefront or by division of amplitude. We will now discuss the interference pattern arising from the superposition of many beams of varying amplitudes.

Such a possibility readily occurs when a parallel glass (or any other dielectric) plate is illuminated by light from an extended source. The incident beam undergoes multiple reflections at both the upper and lower surfaces of the glass plate. Consequently, a large number of reflected and transmitted beams of successively diminishing amplitudes are produced on the two sides of the plate. These multiple reflected (and transmitted) beams move parallel to each other. When these beams are focussed by a suitable lens on its focal plane, an interference pattern is observed.

The amplitudes of reflected and transmitted components at an interface between two mediums is given by the Stokes relations (Sec. 4.1). Thus multiple reflections and transmissions lead to beams of continuously decreasing amplitudes. In Fig. 4.12, we have shown the amplitudes of a few of the reflected and transmitted beams, when a beam of amplitude E_0 is incident at angel θ_0 on the face *AB* of the parallel dielectric plate *ABCD*. $(r\ ,\ t)$ and $(r',\ t'\ )$ denote coefficients of reflection and transmission at interfaces *AB* and *CD* respectively.

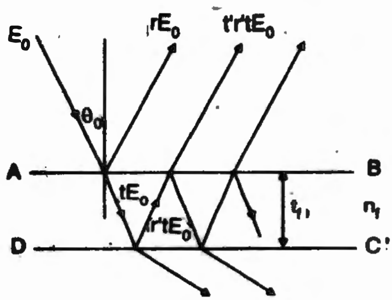

Fig. 4.12

4.7 FABRY PEROT INTERFEROMETER

Fabry Perot Interferometer is a high resolution interferometer based on the principle of multiple beam interference described in the previous section. It is used for precise wavelength measurements, analysing two very close wavelengths (hyperfine spectral analysis), determining refractive indices of gases, and calibration of standard meter scale in terms of wavelengths, etc.

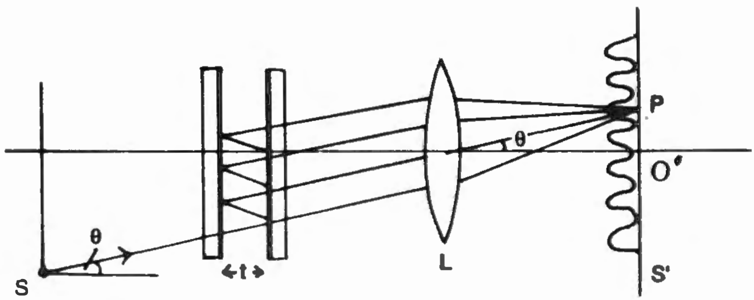

Fig. 4.13

The interferometer consists of two plane glass plates kept strictly parallel to each other so that they enclose a parallel 'plate' of air between them. The inner surfaces of glass plates are polished with highly (80%) relfecting metal film of (of silver or aluminium); these surfaces are made extremely flat, with a flatness of the order of 1/20 to 1/100 of the wavelength. The multiple reflections which occur within the 'air plate' are responsible for the working of interferometer (see Fig. 4.13).

The glass plates themselves are not of uniform thickness; the outer surfaces of these plates are inclined at a small angle(~0.1°) with respect to inner surfaces. This is done to eliminate any (equal inclination) interference fringes that might arise from glass plates themselves (if these were parallel). One of these two plates is kept fixed while the other can be moved with the help of a precise micrometer screw so that the thickness of the air plate can be varied. Sometimes, both the plates are fixed; the instrument is then called a *Fabry Perot Etalon*.

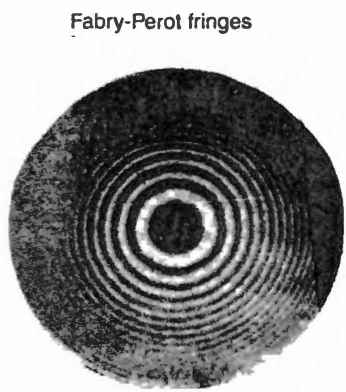

Fig. 4.14