$$\frac{Vd}{\nu} = [(LT^{-1}) \times L \times (L^2T^{-1})^{-1}] = 1$$

Thus the above analysis has collapsed the five original variables n, d, V, ρ and ν into two compound variables, both of which are non-dimensional. This has two advantages: (i) that the values obtained for these two quantities are independent of the consistent system of units used; and (ii) that the influence of four variables on a fifth term can be shown on a single graph instead of an extensive range of graphs.

It can now be seen why the index f was left unresolved. The variables whose indices were resolved appear in both dimensionless groups, although in the group nd/V the density ρ is to the power zero. These repeated variables have been combined in turn with each of the other variables to form dimensionless groups.

There are certain problems, e.g. the frequency of vibration of a stretched string, in which all the indices may be determined, leaving only the constant C undetermined. It is, however, usual to have more indices than equations, requiring one index or more to be left undetermined as above.

It must be noted that, while dimensional analysis will show which factors are not relevant to a given problem, the method cannot indicate which relevant factors, if any, have been left out. It is, therefore, advisable to include all factors likely to have any bearing on a given problem, leaving out only those factors which, on *a priori* considerations, can be shown to have little or no relevance.

1.2 The relevant properties of a fluid

1.2.1 The forms of matter

Matter may exist in three principal forms, solid, liquid or gas, corresponding in that order to decreasing rigidity of the bonds between the molecules of which the matter is composed. A special form of a gas, known as a *plasma*, has properties different from those of a normal gas and, although belonging to the third group, can be regarded justifiably as a separate, distinct form of matter.

In a solid the intermolecular bounds are very rigid, maintaining the molecules in what is virtually a fixed spatial relationship. Thus a solid has a fixed volume and shape. This is seen particularly clearly in crystals, in which the molecules or atoms are arranged in a definite, uniform pattern, giving all crystals of that substance the same geometric shape.

A liquid has weaker bonds between the molecules. The distances between the molecules are fairly rigidly controlled but the arrangement in space is free. A liquid, therefore, has a closely defined volume but no definite shape, and may accommodate itself to the shape of its container within the limits imposed by its volume.

A gas has very weak bonding between the molecules and therefore has neither a definite shape nor a definite volume, but will always fill the whole of the vessel containing it.

A plasma is a special form of gas in which the atoms are ionized, i.e. they have lost one or more electrons and therefore have a net positive electrical charge. The electrons which have been stripped from the atoms are wandering free within the or

$$\Delta p = \rho g \Delta h \tag{1.42}$$

Then by measuring Δh and using the known value of ρ it is possible to calculate the pressure difference Δp, taking care to use consistent units.

Example 1.1 On a certain day the barometric pressure is found to be 750 mm of mercury. A U-tube, filled with alcohol of relative density 0·82, has one limb connected to a point on a model wing in a wind-tunnel and the other limb is open to the atmosphere. The liquid level in the first limb is 25 mm higher than that in the second limb.

Calculate (a) the pressure difference between the point on the model and the atmosphere and (b) the absolute pressure at that point. The relative density of mercury is 13·6 and the density of water is 1000 kg m^{-3}.

(a) The pressure difference supports a column of 0·025 m of the liquid. The pressure difference is, therefore

$$\begin{aligned}\Delta p = \rho g \Delta h &= (0{\cdot}82 \times 1000) \times 9{\cdot}807 \times 0{\cdot}025 \\ &= 201\ \mathrm{N\,m^{-2}}\end{aligned}$$

Since the liquid has been drawn higher in the limb connected to the model, the air pressure in that limb must be lower than atmospheric pressure.

(b) $$\begin{aligned}\text{Barometric pressure} &= (13{\cdot}6 \times 1000) \times 9{\cdot}807 \times 0{\cdot}75 \\ &= 100\,050\ \mathrm{N\,m^{-2}}\end{aligned}$$

Therefore the absolute pressure at the point on the model is

$$100\,050 - 201 = 99\,850\ \mathrm{N\,m^{-2}}$$

In aeronautics it is often necessary to measure pressure differences corresponding to $\frac{1}{4}$ mm of water or less. For such small pressure differences the simple U-tube is not sufficiently sensitive and various other forms of manometer have been developed to measure these small differences. Most of these are based on the U-tube and differ only in the method used to give the increased sensitivity. Two principal methods are:

(i) Making the fluid displacement to be measured greater than the displacement in a simple U-tube. A common example of this in aeronautics is the inclined tube manometer. The fluid displacement is increased by the factor cosec θ, θ being the angle of slope to the horizontal.

(ii) Using some optical arrangement to magnify the actual measurement. Examples include the Casella and Betz manometers.

Many instruments use alcohol or similar fluids with densities less than that of water to obtain larger displacements. An alternative type of instrument is the electrical pressure transducer.

1.3.3 The atmosphere

Since the atmosphere may be regarded as an expanse of fluid (air) substantially at rest, hydrostatic theory may be used to calculate its macroscopic properties. The atmosphere is a mixture of gases of which oxygen and nitrogen are the main constituents, but it also contains small amounts of other gases, including hydrogen and helium and the rare inert gases argon, krypton, neon, etc. Over the range of altitudes involved in conventional aerodynamics the proportion of the constituents

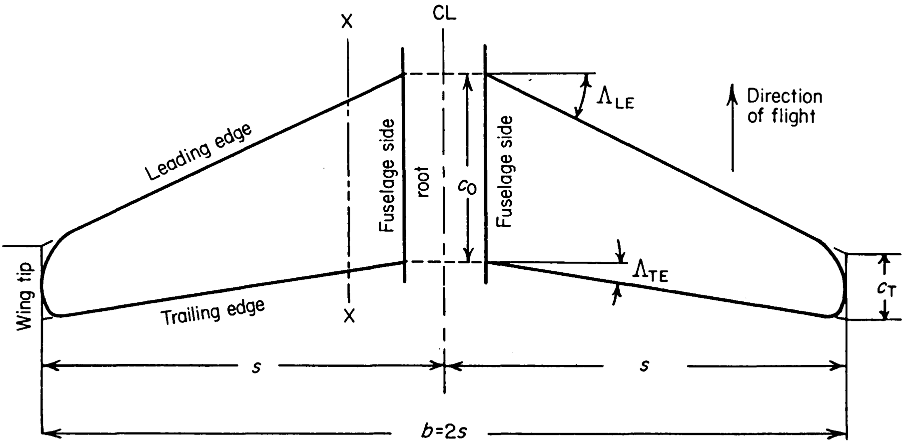

Fig. 1.8 Wing planform geometry

Chords

The two dimensions c_T and c_0 are the tip and root chords respectively; with the alternative convention, the root chord is the distance between the intersections with the fuselage centre-line of the leading and trailing edges produced. The ratio c_T/c_0 is the taper ratio λ. Sometimes the reciprocal of this, namely c_0/c_T, is taken as the taper ratio. For most wings $c_T/c_0 < 1$.

Wing area

The plan-area of the wing including the continuation within the fuselage is the gross wing area, S_G. The unqualified term wing area S is usually intended to mean this gross wing area. The plan-area of the exposed wing, i.e. excluding the continuation within the fuselage, is the net wing area, S_N.

Mean chords

A useful parameter, the standard mean chord or the geometric mean chord, is denoted by $\bar{c}$, defined by $\bar{c} = S_G/b$ or S_N/b. It should be stated whether S_G or S_N is used. This definition may also be written as

$$\bar{c} = \frac{\int_{-s}^{+s} c\,\mathrm{d}y}{\int_{-s}^{+s} \mathrm{d}y}$$

where y is distance measured from the centre-line towards the starboard (right-hand to the pilot) tip. This standard mean chord is often abbreviated to SMC.

be some characteristic length, e.g. the wing span, of the full-size aircraft and B_m the corresponding length on the model.
(i) Full-size aircraft in flight

$$\mu = \mu_0\left(1+\frac{5\cdot 1}{273}\right)^{3/4} \text{ by Rayleigh's Law} = 1\cdot 014\mu_0$$

$$\text{Flight speed } V = 35 \text{ EAS m s}^{-1}$$
$$= 35(0\cdot 862)^{-1/2} = 37\cdot 7 \text{ TAS m s}^{-1}$$

$$\text{Reynolds number } Re = VB_f/\nu = VB_f\rho/\mu$$
$$= \frac{37\cdot 7B_f\, 0\cdot 862\rho_0}{1\cdot 014\mu_0} = 32\cdot 1B_f\frac{\rho_0}{\mu_0}$$

(ii) Model under test

$$\mu = \mu_0\left(1+\frac{15}{273}\right)^{3/4} = 1\cdot 0412\mu_0$$

Since the pressure in the tunnel is 22 atmospheres while the temperature is 15°C, i.e. standard, it follows that the density will also be 22 times the sea-level standard value. Thus

$$Re = \frac{30B_m 22\rho_0}{1\cdot 0412\mu_0} = 634B_m\frac{\rho_0}{\mu_0}$$

For dynamic similarity, these two values of the Reynolds number must be equal, i.e.

$$32\cdot 1B_f = 634B_m$$

$$\frac{B_f}{B_m} = 19\cdot 8$$

Thus the model scale should be 1/19·8 of full size. For low speeds, Eqn (1.68) becomes:

$$F = \rho V^2B^2g(Re)$$

By ensuring equality of the Reynolds numbers, Re, it is automatically ensured that the function $g(Re)$ has the same value for both model and full-size aircraft. Then, if L_f is the lift of the aircraft and L_m the lift of the model:

$$\frac{L_m}{L_f} = \frac{\rho_m}{\rho_f}\left(\frac{V_m}{V_f}\right)^2\left(\frac{B_m}{B_f}\right)^2 = \frac{22}{0\cdot 862}(0\cdot 796)^2\left(\frac{1}{19\cdot 8}\right)^2 = 0\cdot 0412$$

Since

$$L_f = 65\,000\text{ N},\ L_m = 65\,000\times 0\cdot 0412 = 2680\text{ N}$$

It should be noted that the model sustains 0·0412 of the lift of the full-sized aircraft, but its wing area is only $(1/19\cdot 8)^2$ of that of the aircraft. Thus the intensity of loading on the model is $0\cdot 0412\times(19\cdot 8)^2$, i.e. 16·2 times that on the aircraft in flight. This points to the need for models used in compressed-air tunnels to be very strong and rigid, to prevent excessive bending which might invalidate the geometric similarity between the aircraft and the scale model.

Example 1.7 An aeroplane approaches to land at a speed of 40 m s^{-1} at sea level. A 1/5th scale model is tested under dynamically similar conditions in a CAT working at 10 atmospheres pressure and 15°C. It is found that the load on the tailplane is subject to impulsive fluctuations at a frequency of 20 cycles per second, due to eddies being shed from the wing-fuselage junction. If the natural frequency of flexural vibration of the tailplane is 8·5 cycles per second, could this represent a dangerous condition?