Motion Under Gravity, Free-fall

When an object falls to the ground under gravitational pull, experiment shows that the object has a constant or uniform acceleration of about 9·8 m s^{-2} or 10 m s^{-2} approximately, while it is falling. The numerical value of this acceleration is usually denoted by the symbol g. Drawn as a vector quantity, g is always represented by a straight vertical line with an arrow on the line pointing downwards, Figure 1.3 (i).

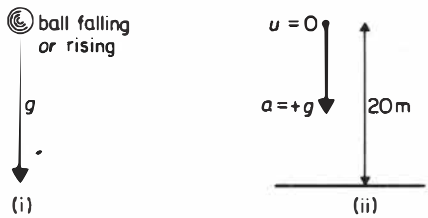


Figure 1.3 *Motion under gravity—free-fall*

Suppose that a ball is dropped from a height of 20 m above the ground, Figure 1.3 (ii). Then the initial velocity $u = 0$, and the acceleration $a = g = 10$ m s^{-2} (approx.). When the ball reaches the ground, $s = 10$ m. Substituting in $s = ut + \frac{1}{2}at^2$, then

$$s = 0 + \tfrac{1}{2}gt^2 = 5t^2$$

$$\therefore\ 20 = 5t^2 \quad \text{or} \quad t = 2\text{ s}$$

So the ball takes 2 seconds to reach the ground.

If a cricket-ball is thrown vertically upwards, it slows down owing to the attraction of the earth. The magnitude of the deceleration is 9·8 m s^{-2}, or g. Mathematically, a deceleration can be regarded as a negative acceleration in the direction along which the object is moving; and so $a = -9{\cdot}8$ m s^{-2} in this case.

Examples on Motion under Free-fall (*Gravity*)

1 A ball is thrown vertically upwards with an initial velocity of 30 m s^{-1}.
Find (i) the time taken to reach its highest point, (ii) the distance then travelled. (Assume $g = 10$ m s^{-2}.)

(*Analysis* (i) We need time t. So we can use $v = u + at$. (ii) We need distance s. So we can use $s = ut + \frac{1}{2}at^2$.)

(i) Here $u = 30$ m s^{-1}, $v = 0$ at highest point since ball is momentarily at rest, $a = -g = -10$ m s^{-2}. From $v = u + at$,

$$0 = 30 + (-10)t \quad \text{or} \quad 10t = 30 \text{ and so } t = 3\text{ s}$$

(ii)

$$\text{Distance } s = ut + \tfrac{1}{2}at^2$$

$$= (30 \times 3) + \tfrac{1}{2} \times (-10) \times 3^2$$

$$= 90 - 45 = 45\text{ m}$$

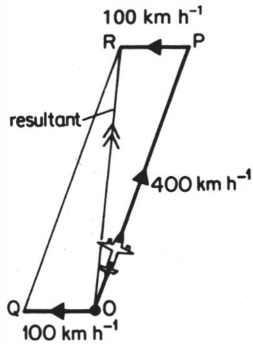


Figure 1.12 *Vector addition*

Subtracting Vectors

In a top-class sprint race, an athlete A may be running at 10 m s^{-1} and another athlete B may be running at 9 m s^{-1} in the same direction. The velocity of A relative to B is then $10 - 9 = 1$ m s^{-1}, which is the difference between the two velocities.

If the two velocities are not in the same direction, we can find the relative velocity again by subtraction, taking into account that we are dealing with vectors.

Suppose that a car X is travelling with a velocity v along a road 30° E. of N., and a car Y is travelling with a velocity u along a road due east, Figure 1.13 (i).

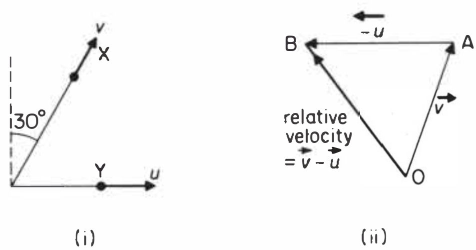
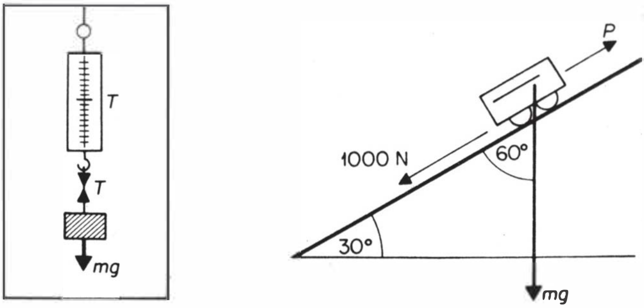


Figure 1.13 *Subtracting vectors*

Arrows above the velocity letters show they are vectors. So the velocity of X relative to Y = difference in velocities = $\vec{v} - \vec{u} = \vec{v} + (-\vec{u})$. Suppose OA represents the velocity, v, of X in magnitude and direction, Figure 1.13 (ii). Since Y is travelling due east, a velocity AB numerically equal to u but in the due *west* direction represents the vector $(-\vec{u})$. The vector sum of OA and AB is OB. So OB represents in magnitude and direction the velocity of X minus that of Y. By drawing an accurate scale diagram of the two velocities, OB can be found in size and direction.



(i) (ii)

Figure 1.16 *Examples on force and acceleration*

(ii) When the lift *descends* with an acceleration of 0·1 m s^{-2}, its weight, 20 N, is now greater than T_1, the new tension in the spring-balance.

$$\therefore \text{ resultant force} = 20 - T_1$$

$$\therefore F = 20 - T_1 = ma = 2 \times 0{\cdot}1$$

$$\therefore T_1 = 20 - 0{\cdot}2 = 19{\cdot}8 \text{ N}$$

(iii) When the lift moves with constant velocity, the acceleration is zero. Since the resultant force is zero, the reading on the spring-balance is exactly equal to the weight, 20 N.

3 *Inclined plane* A car of mass 1000 kg is moving up a hill inclined at 30° to the horizontal. The total frictional force on the car is 1000 N, Figure 1.16 (ii).

Calculate the force P due to the engine when the car is
(a) accelerating at 2 m s^{-2},
(b) moving with a steady velocity of 15 m s^{-1}.

(*Analysis* (a) Use $F = ma$. (b) There are three forces on the car—its weight mg, P and the frictional force. (c) Since the car is moving on an incline, we need to find the component of the weight mg, down the incline.)

(a) Weight of car = mg = 10 000 N
Component downhill = $mg \cos\theta$ = 10 000 cos 60° = 5000 N

So resultant force uphill, $F = P - 5000 - 1000$

From $F = ma$,

$$P - 5000 - 1000 = 1000 \times 2 = 2000$$

$$P = 8000 \text{ N}$$

(b) Since velocity is steady, acceleration $a = 0$
So resultant force $F = 0$

Then $P = 5000 + 1000 = 6000$ N

Terminal Velocity

If we swim or drive a car fast, we can feel the frictional force or resistance due to motion through water or to air. The size of F increases with the velocity v

Force due to Rotating Helicopter Blades

When helicopter blades are rotating, they strike air molecules in a downward direction. The momentum change per second of the air molecules produces a *downward* force and by the Law of Action and Reaction, an equal *upward* force is exerted by the molecules on the helicopter blades. This upward force helps to keep the helicopter hovering in the air because it can balance the downward weight of the machine. This is illustrated in the example which follows.

Example on Rotating Helicopter Blades

A helicopter of mass 500 kg hovers when its rotating blades move through an area of $30\ m^2$ and gives an average speed v to the air.

Estimate v assuming the density of air is $1{\cdot}3\ kg\ m^{-3}$ and $g = 10\ N\ kg^{-1}$.

(*Analysis* (i) The reaction of the downward force on the air = weight of helicopter, (ii) downward force = momentum change per second of air swept down, (iii) mass of air per second moving downwards = volume per second × density = area swept by blades × velocity of air × density.)

Volume of air per second moving downwards = area × velocity $v = 30v$

So mass of air per second downwards $= 30v \times 1{\cdot}3 = 39v$

∴ momentum change per second of air = mass per second × velocity change

$$= 39v \times v = 39v^2$$

So reaction force upwards $= 39v^2$ = helicopter weight 5000 N

$$v^2 = \frac{5000}{39}$$

$$v = \sqrt{\frac{5000}{39}} = 11\ m\ s^{-1}\ \text{(approx.)}$$

At lift-off, the fuselage of the helicopter would turn round the opposite way to the rotation of the blades, from Newton's law of Action and Reaction. A vertical rotor on the tail provides a counter-thrust.

You should know:

1. **Momentum = mass × velocity.**
   Impulse = force × time ($F.t$) = mass × velocity change
   Momentum or impulse unit: N s
2. **Newton's second law: force, F = rate of change of momentum ($F = d(mv)/dt$)**
3. ***Mass constant:* use $F = ma$ to find force**
   ***Mass varying:* to find force, use F = mass per second × velocity change**
4. **Newton's third law: Action and reaction forces are equal and opposite. The law applies to forces between objects in contact *or* at a distance from each other, as in a gravitational field (or magnetic or electric field)**